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G. S. Kupriyanova, S. V. Molostova, Zh. Yu. Nesterova

PRACTICAL PHYSICS

A physics textbook for students of General Medicine

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Reviewers

Dr hab. Prof. M. A. Nikitin, Institute of High Technology, IKBFU;
Dr A. V. Pets, Associate Professor, Institute of High Technology, IKBFU

Kupriyanova, G.S.

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The *Practical Physics* handbook is designed for students of higher education institutions specialising in General Medicine. It serves as a guide for completing laboratory practical work in physics and provides reference materials covering the main sections of the discipline. For self-assessment, the book includes lecture questions, test tasks and practical exercises for all sections. Using this aid alongside primary information sources enables students to optimise their study of physics.

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AUTHORS' PREFACE

Physics is a natural science that studies the properties and structure of matter, the laws governing its motion and transformation and the general principles of natural phenomena. It is grounded in experience and observation. To characterise the qualities and properties of a material object or phenomenon, physics employs physical quantities and corresponding units of measurement. General patterns are identified through experiments. The proper design of an experiment, the selection of equipment with appropriate accuracy and the use of correct measurement techniques are essential for determining physical quantities and constructing adequate models of natural phenomena. One of the central tasks of physics is to ensure precise experimental results and accurate measurements of physical quantities.

This handbook includes an **introduction, descriptions of laboratory** in kinematics, dynamics, electricity and optics and reference material that includes the basic laws of physics and key formulas in the **Support Materials**. Sections 1, 2 and 3 of the Introduction examine the nature of measurement, the different types of measurements and how to estimate measurement error correctly. Sections 4 and 5 discuss methods for calculating errors in indirect measurements and the least squares method for determining functional relationships between physical quantities. The second part of the handbook presents descriptions of 12 laboratory and provides methods for calculating the measured physical quantities. Data required to complete the laboratory exercises are provided in the Support Materials.

I. INTRODUCTION

Section 1

Measurements. Classification of measurements

Measurement is the experimental determination of the value of a physical quantity using specialised technical means, with the result expressed in units of that quantity.

Measurements are categorised as **direct** or **indirect**. A **direct measurement** is one in which the value of a quantity is read from the scale of an instrument calibrated in the appropriate units. In this process, the value of a physical quantity is obtained by comparing the unknown quantity directly with a standard.

Direct measurements do not require mathematical calculations to obtain a physical quantity. The only factor to consider is the scale division value.

The value of a physical quantity can be determined using the equation for direct measurement: $y = cx$, where y is the measured quantity, c is the scale division value in units of the measured quantity, and x is the reading of the instrument in scale divisions.

Examples of direct measurements include measuring the length of an object with a caliper or micrometer, measuring current with an ammeter, measuring voltage with a voltmeter or measuring temperature with a thermometer.

Thus, direct measurement involves using specialised instruments designed either to measure a physical quantity directly or to convert certain signals into electrical characteristics.

An indirect measurement is a measurement in which the result is determined from direct measurements of quantities that are related to the measured quantity by a functional relationship.

The equation of indirect measurement has the following form

$$F = f(x_1, x_2, \dots, x_n), \quad (\text{I.1})$$

where F is the desired value, expressed as a function of x_1, x_2, \dots, x_n , measured by the direct method. An indirect measurement is one in which the result is calculated using formula (I.1).

An example of indirect measurements is determining the radius of a ball. We can measure the circumference of a ball L with a tape measure and then calculate the radius R using the circle formula $2\pi R=L$. Another example is measuring the volume of a cube: by measuring the side D with a caliper, the volume can be calculated with the formula $V=D^3$.

Devices for direct measurement include timers, stopwatches, micrometers, calipers, ammeters (Fig. I.1), voltmeters, thermometers and pressure meters.

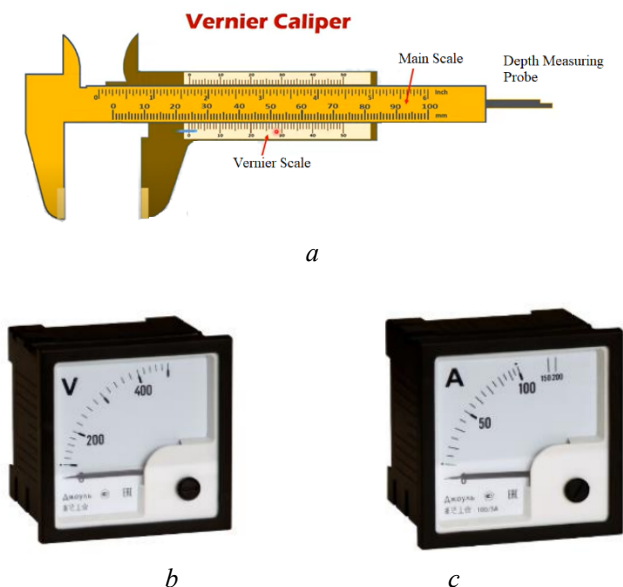


Fig. I.1. Instruments for direct measurements:
 a — vernier caliper; b — voltmeter; c — ammeter

Section 2

Types of devices and sensors. Device characteristics

Sensors

Sensors convert the measured parameter into an electrical signal. They are classified into two main types: **active** and **passive**.

Active sensors generate electrical current or voltage in direct response to the measured parameter, such as physical impact, thermal energy, pressure, motion or electromagnetic and acoustic energy. The amplitude or frequency of the signal produced by the sensor is proportional to the measured parameter.

Passive sensors change their electrical characteristics, such as resistance, capacitance or inductance, in response to the measured parameter. They usually require an external source of electrical energy. Many biosignals — blood pressure, body temperature, blood flow and cerebrospinal fluid pressure — do not inherently possess electrical potential or voltage.

Monitoring is possible only with sensors that convert the signal into a physical quantity or another form of electrical signal.

Main characteristics of sensors

The **sensitivity** S of a sensor is the minimum input x of a physical parameter that produces a detectable change in the output signal. It is defined as the ratio of the incremental change in the output signal to the incremental change in the input signal, i. e.

$$S = \frac{\Delta y}{\Delta x}. \quad (\text{I.2})$$

In some sensors, sensitivity is defined as the change in the input parameter required to produce a standardised change in the output signal. In others, it is defined as the change in output voltage corresponding to a given change in the input parameter (I.2).

The **measured range (MR)** of a sensor or device is the maximum and minimum values of the applied parameter that the sensor or device can measure. It is defined as the difference between the maximum input signal and the minimum input signal:

$$x_{\max} - x_{\min} = MR. \quad (I.3)$$

The **accuracy of a sensor** is the maximum difference between the actual value (as measured by a primary or reliable secondary standard) and the value indicated by the sensor.

The accuracy of a sensor indicates how close a measurement is to the true value of the measured variable. It determines how near an experiment comes to the ‘correct answer’. For example, if a body temperature is known to be 36 °C, the thermometer is considered accurate if it reproduces this value.

Accuracy is determined by the equation:

$$\varepsilon_a = \frac{x_m - x_{true}}{x_{true}}, \quad (I.4)$$

where x_{true} stands for the true value and x_m for the measured value.

The accuracy of a sensor can be expressed as either a percentage of full scale or in absolute values. The difficulty in assessing accuracy lies in the uncertainty of the ‘true value’, which may be a standard value or one obtained from multiple experiments.

Resolution is the smallest incremental change in the input signal producing a detectable change in the output signal. It is often expressed as a percentage of the measured range (I.3), $MR = \Delta x_{\max} - \Delta x_{\min}$. For a detectable output signal Δy , if the minimum change in the input signal is Δx_{\min} , the maximum resolution is:

$$R_{max} = \frac{x_{min}}{MR}. \quad (I.5)$$

The **response time** is the time required for a sensor’s output signal to change from its previous state to its final value within an acceptable range of the correct new value. Sensors do not alter their output immediately when the input parameter changes; instead, the output transitions to the new state over a period known as the response time.

Analogue and digital devices

When planning an experiment, it is crucial to choose the appropriate type of instruments for inputting, converting and measuring the signal. There are two types of devices: analogue and digital. **Analogue devices** are used to process and record **analogue signals**, which are continuously varying and can be described by a continuous function.

Digital devices use discrete signals generated by devices that can only assume two states, 1 and 0 , with the **digital signal** represented as a sequence or discrete set of numbers.

Analogue devices are suitable for experiments requiring precise measurement or control of parameters that may change continuously, such as temperature, pressure, flow, current, speed or level. They can measure or convert a wide range of values corresponding to physical phenomena. For example, an analogue temperature sensor can produce a voltage that varies in proportion to the temperature it detects.

These devices record more accurate and detailed information than digital devices and offer greater flexibility and compatibility with different types of sensors, but they are more expensive and complex. A disadvantage of analogue devices is their susceptibility to noise and interference, which can affect measurement quality and accuracy. Due to their high sensitivity, analogue devices require more extensive noise suppression and calibration than digital devices.

Various natural sounds — thunder, lightning, the barking of dogs and the singing of birds — are analogue signals. These signals may have a certain duration or even discontinuities, as in a nightingale's trill, yet they represent continuous functions rather than discrete numbers.

Digital devices also offer several advantages. They are cheaper and simpler than analogue devices, more reliable and robust, less

susceptible to noise and interference, require fewer matching devices and calibration, and interface easily with computers. However, digital devices may provide less information than analogue devices, process only simple and linear signals, and have limited compatibility with different types of sensors.

Our mobile phones and computers, which contain various digital logic devices and microprocessors, are purely digital devices. Nevertheless, an analogue signal can be converted into a digital signal by dividing it into segments and digitising each part. **Analogue-to-digital converters** are used to convert the original signal into digital form.

In our laboratory, you will use both analogue devices, such as voltmeters and ammeters, and digital devices, such as digital multimeters and oscilloscopes, to measure currents and voltages.

Section 3 Error of direct measurement

Random error of direct measurement

The main task of an experiment is to determine the true value x_0 of a physical quantity. However, when a physical quantity is measured using a device, the process is accompanied by a measurement error. The true value x_0 can be determined if the complete set of all measured values x is known. Measurement error is determined by several factors, including random errors and the systematic errors of the devices used to measure the observables.

Let us consider how to estimate the **random error of a direct measurement**. Given the finite number of observations in the sample, the experiment must determine a value x close to x_0 , known as the estimate of the true value. It is also necessary to specify the interval within which the true value x_0 lies with a given probability P . This interval is called the **confidence interval**, and P is the **confidence probability**.

Let us assume that the confidence interval has been determined, within which the probability of containing the true value of the desired quantity is known. This enables us to choose the instruments and evaluate their accuracy. Let us also assume that a series of N measurements of the desired quantity has been carried out.

As an estimate of the true value based on the data, the arithmetic mean of the sample observations is used, calculated by the formula:

$$\bar{x} = \frac{1}{N} \sum_{i=1}^n x_i = \frac{x_1 + x_2 + \dots + x_n}{N}. \quad (\text{I.6})$$

This value is called the **sample mean (I.6)**.

The greater the number of measurements of the desired value, the more accurate the result.

The **average value x is also a random variable**, and if we repeat the process of determining it several times, we obtain a sample of average values x : x_1, x_2, \dots, x_b , which will also differ randomly, though the spread of the average values will be significantly smaller than the spread of the individual observations in each sample.

To determine the confidence interval P , it is necessary to know the distribution of the mean values (I.6) $f(x)$ around x_0 . Knowing the form of $f(x)$ makes it possible to construct an interval within which the true value x_0 lies with probability P . It should be noted that after determining the sample mean \bar{x} , the range of values in which the measured value lies can be narrowed, increasing the confidence probability P . Furthermore, it may be possible to use more sensitive devices with higher measurement accuracy.

A quantitative measure of the scatter of experimental data, reflecting the quality of the experiment, is the **dispersion σ^2** . Typically, only a finite number of experiments is conducted, resulting in a limited sample size n . From this sample, the sample variance S^2 can be calculated,

$$S^2 = \sum_{i=1}^n \frac{(x_i - \bar{x})^2}{N}, \quad (\text{I.7})$$

where \bar{x} is the sample mean, and x_i is the current value of the experimental results from the first to the n^{th} measurement. The sample variance S^2 (I.7) is an estimate of the true variance σ^2 . As the number of samples increases to infinity (as $n \rightarrow \infty$), the sample variance approaches the variance, $S^2 \rightarrow \sigma^2$ (dispersion).

Random variables whose values depend on many independent factors follow the **normal, or Gaussian, distribution**. The **probability density function $f(x)$** of the normal distribution has the form:

$$f(x) = \frac{1}{\sigma_x \sqrt{2\pi}} e^{-(x-x_0)^2/2\sigma_x^2}, \quad (\text{I.8})$$

where x is a random value of X . The parameter x_0 determines the centre of the distribution, and the dispersion σ_x^2 determines the shape and width of the distribution density curve.

The factor $\frac{1}{\sigma_x\sqrt{2\pi}} = f(x_0)$ defines the height of a Gaussian curve (Fig. I.2) and can be calculated from the normalisation condition, which is satisfied as follows:

$$\int_a^b dP(x) = \int_a^b f(x)dx = 1. \quad (\text{I.9})$$

The normalisation condition for the probability density function $f(x)$ requires that the area under the probability function curve (I.9) is always equal to one. This has a clear physical meaning: the total probability of locating the true value is one.

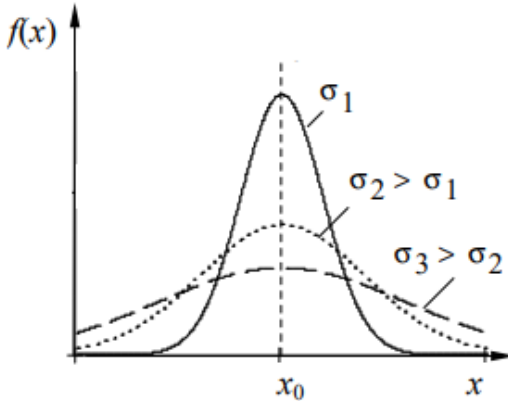


Fig. I.2. Gaussian distribution functions for dispersion values of variance

The graph (Fig. I.2) shows that the smaller the variance σ_x , the narrower the Gaussian distribution and the closer the measured value is likely to be to the correct value. The English mathematician William Sealy Gosset, who published under the pseudonym Student, found that the **random confidence error of measurement results depends on the dispersion**. He established that the **random confidence error of a measurement result should be calculated using the formula:**

$$\Delta x = t_{P,N} S_x. \quad (\text{I.10})$$

In formula (I.10), $t_{P,N}$ is Student's coefficient, which is determined by the confidence probability P and the sample size N . The values x and S_x , proportional to the variance, are calculated using the following formula:

$$S_x = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N(N-1)}}. \quad (\text{I.11})$$

The parameter S_x is called the **sample standard deviation of the mean**.

Random measurement error defines the **confidence interval** within which the measured quantity lies. The values of Student's coefficients, denoted as $t_{p,N}$, are calculated using probability theory and are determined by the number of observations N and the confidence probability P . The confidence probability is usually taken as 0.9 or 0.95. Table I.1 presents the corresponding values of Student's coefficients for the specific case where $P=0.95$ (95%).

The table clearly shows that the greater the number of measurements, the smaller the Student's coefficient.

Table I.1

Values of Student's $t_{p,N}$ at $P=95\%$

N	2	3	4	5	6	7	8	9	10	100
$t_{P,N}$	12.7	4.3	3.2	2.8	2.6	2.5	2.4	2.3	2.3	2.0

Formula (I.10) allows estimation of the absolute error, defined as the difference between the instrument reading and the true value. **Absolute error** is expressed in units of the measured value. Alongside absolute error, it is important to assess the **relative measurement error**, which is the ratio of the absolute error to the measurement result, given by the equation:

$$\varepsilon_a = \frac{x_m - x_t}{x_t} 100\%, \quad (\text{I.12})$$

where x_t is the true value, x_m is the measured value.

The relative measurement error reflects the accuracy of the experiment. It is not always possible to determine the percentage difference in the laboratory, as the 'true value' is often unknown. Formula (I.12) can be compared with formula (I.4), which uses the exact value. If the exact value is known or can be calculated in advance, it can be used to verify the accuracy of the experiment. In general, a relative measurement error greater than 10% indicates that the experiment produces an insufficiently precise value for the observed quantity. In such cases, the source of the problem must be identified and new data collected.

Systematic error. Instrument accuracy class

Thus, when performing measurements, it is necessary to consider not only the random measurement error but also the type of device used and its accuracy. Measuring instruments are classified as follows, depending on their measurement accuracy:

1. The most precise instruments are devices and installations for high-precision measurements. An example is interferometers, which are used to measure the length and shape of optical components with nanometer precision.

2. The second group consists of less precise devices, such as microscopes, comparators and optimeters.

3. The third group includes even less precise devices, such as dial indicators, verniers, calipers and rulers with vernier scales. In our lab, we will primarily use devices from the third group. Second-group devices, such as microscopes, are used to study microorganisms, whereas interferometers from the first group are required to determine the wavelength of light.

Digital and analogue devices provide different measurement accuracies, requiring distinct approaches to calculating measurement errors. The measurement accuracy of a device is determined by its accuracy class.

1. The **accuracy class** is the maximum **relative error** of a measurement result, expressed as a percentage, ϵ_a . It is indicated by

a number on the front panel of the device or specified in the device description. Knowing the accuracy class allows estimation of the **absolute error**, which in this case is $\Delta x = \varepsilon_a x / 100$, where x is the reading of the physical quantity on the device scale in the corresponding units of measurement.

2. If the accuracy class is indicated simply by a number, it represents the maximum error of the device (error margin), expressed as a percentage of the maximum scale reading K of the device.

Consider, for example, using a milliammeter to measure current and estimate the maximum measurement error.

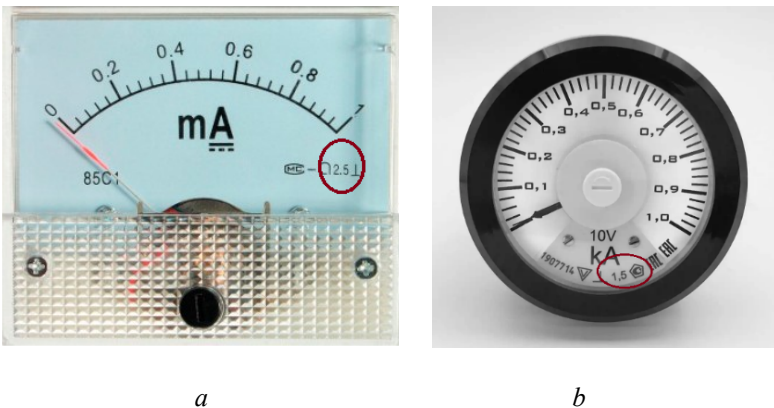


Fig. I.3. A milliammeter (*a*) and a device for measuring current and voltage (*b*)

Figure I.3, *a* shows a milliammeter. The accuracy class of the device is indicated on the instrument panel (marked by the red circle). It can be seen that the accuracy class is 2.5, and the maximum scale value of the device is 1 mA. The maximum **systematic** absolute error in this case is $\Delta x_{sys} = 2.5 \times 1 \text{ mA} / 100 = 0.025 \text{ mA}$.

In Figure I.3, *b*, a device for measuring both current and voltage is shown. The accuracy class is 1.5 (see the red circle). If the device is used as a voltmeter to measure voltage, the **systematic**

absolute error is $\Delta x_{sys} = 1.5 \times 10 / 100 = 0.15 \text{ V}$, as the full voltage scale corresponds to 10 V. If the device is used as an ammeter to measure current, the full scale is 1 kA, giving a **systematic** absolute current error of $\Delta x_{sys} = 1.5 \times 1 / 100 = 0.015 \text{ kA}$.

3. If the accuracy class of an analogue (pointer) instrument is not specified, its maximum absolute error Δx_{sys} is taken as half the value of the instrument's smallest scale division. The smallest scale division of such a device usually corresponds to the instrument's own error. Therefore, attempting to read fractions of the smallest division from the scale is impractical and does not reduce the systematic instrumental error.

4. For a **digital measuring device** with an unknown accuracy class or without a specified formula for calculating the error, the maximum absolute error Δx_{sys} is estimated as the unit of the smallest digit of the indicator in a single reading, or as the unit of the last stable (non-blinking) digit during continuous measurements. For example, the maximum error of an electronic watch is $\Delta x_{sys} = 1 \text{ min}$, and the maximum error of a **stopwatch** is $\Delta x_{sys} = 1 \text{ s}$.

Total measurement error. Addition of random and systematic errors

One should consider that the systematic error introduced by the devices causes a shift θ in the sample mean \bar{x} of the observation results.

$$\bar{x}' = \frac{1}{N} \sum_i (x_i + \theta) = \frac{1}{N} \sum_i x_i + \frac{1}{N} \sum_i \theta = \bar{x} + \theta. \quad (\text{I.13})$$

Therefore, the total error of the measurement result is calculated by the formula below:

$$\Delta \bar{x} = \sqrt{\Delta x^2 + \Delta \theta_x^2}, \quad (\text{I.14})$$

where $\Delta \theta_x$ is the systematic error of the device used to measure the physical quantity x . Note that this formula (I.14) is only used when making direct measurements.

Processing direct measurement data using a sample

Let us consider the experimental procedure, the methods of data recording, and the algorithm for processing direct measurement data using a sample.

1. Now let us consider an experiment aimed at determining the acceleration of bodies in an Atwood machine. To this end, we measure the time required for the weights to move between two points, the distance between which is already known. First, it is necessary to select a device capable of accurately measuring a short time interval. Both analogue and digital watches may be used. A conclusion should be drawn on whether the chosen device is suitable for the measurements. Let us select a high-precision stopwatch.

2. The next step is to determine the accuracy class of the chosen device. If the accuracy class is not specified, the absolute systematic error of the device is taken as the value of its smallest scale division. Let us assume we are using a stopwatch with an absolute systematic error of 0.01 s.

3. A physical quantity is measured multiple times to increase measurement accuracy.

4. Let us assume that we have measured the physical quantity seven times.

5. Next, the obtained experimental data $x_{i(s)}$ are entered into Table I.2.

Let us process the data and find random and total errors.

6. Let us analyse the experimental data obtained in the first row (Table I.2) and review the resulting values. It is evident that the value 6.2s differs significantly from the others. Let us exclude obvious errors from the sample by removing repeated measurements or those that deviate significantly from the average.

7. Let us calculate the sample mean

$$\bar{x} = (x_1 + x_2 + x_3 + x_4 + x_5 + x_6) / 6.$$

We discard the last value, 6.2s. In our case $\bar{x} = 5.89$ seconds.

Table I.2

An algorithm for processing the experimental data

x_i (s)	5.90	5.83	5.89	5.91	5.92	5.90	6.20
x_i (s)	5.90	5.83	5.89	5.91	5.92	5.90	—
\bar{x} (s)	5.89						
$x_i - \bar{x}$ (s)	0.0083	-0.0617	-0.0017	0.0183	0.028	0.0083	
$\sum_{i=1}^N (x_i - \bar{x})$	0.0005						
$(x_i - \bar{x})^2 / (6 \times 5)$	6.94×10^{-5}	0.0038	2.78×10^{-6}	0.00034	0.0008	6.94×10^{-5}	
$\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{6 \cdot 5}$	0.000169						
$S_x = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N(N-1)}}$	0.013						

8. The next step is calculating the deviation from the average. The sum of deviations from the mean, including their signs, should be zero. Let us check by adding all the values in the row. Our result is 0.0005. This value must be rounded to the second significant figure, taking into account the accuracy of the instrument. A stopwatch cannot provide accuracy greater than its systematic error; therefore, our result is considered zero, which is a good outcome.

The sum of deviations from the mean, including their signs, should be zero. Let us check by adding all the values in the row. The result is 0.0005. This value must be rounded to the second significant figure, taking into account the accuracy of the instrument. A stopwatch cannot provide accuracy greater than its systematic error. Therefore, our result is considered zero, which is a good outcome.

9. The next stage involves calculating the sample standard deviation of the mean. Let us find the parameter S_x using the formula (I.12):

$$S_x = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N(N-1)}}$$

The calculation yields $S=0.013$ seconds.

10. We can now estimate the random error using Student's formula. Let us set the confidence probability P to 95 % and use this value to calculate Student's coefficient from Table I.1. In our case, the number of samples N is 6, so $t_{p,N}=2.6$.

According to Student's formula (I.10), the **random error of the measurement result** can be calculated as

$$\Delta x_{\text{ran}} = t_{p,N} S_x.$$

Our result is:

$$\Delta x_{\text{ran}} = 2.6 \times 0.013 \text{ (s)} = 0.0338 = 0.03\text{s}.$$

Determine the upper bound of the instrument error θ_x based on the instrument's accuracy class or systematic error. In our case, $\theta_x=0.01$ s.

11. Next, using formula (I.14), we calculate the total absolute error of the measurement result:

$$\Delta \bar{x} = \sqrt{\Delta x^2 + \Delta \theta_x^2} = 0.0316\text{s}. \quad (\text{I.15})$$

12. To calculate the relative error, let us determine the ratio of the total absolute error to the mean value.

$$\varepsilon_a = \frac{\Delta \bar{x}}{\bar{x}} \times 100 \% = 0.53 \%. \quad (\text{I.16})$$

13. Let us express the final result as:

$$x = \bar{x} \pm \Delta \bar{x} = (5.89 \pm 0.03)\text{s}. \quad (\text{I.17})$$

The total absolute error (I.15) is $\Delta \bar{x} = 0.03\text{s}$.

The relative error (I.16) is $\varepsilon_a = 0.005$, $\varepsilon_a = 0.5 \%$.

This algorithm should be used to calculate the errors for each quantity measured in the experiment.

* Please note that the obtained absolute error of 0.0316s is rounded to 0.03s, which is recorded as the final result (I.17).

Section 4

Error of indirect measurements

Indirect measurement is the determination of a physical quantity based on measurement data obtained using devices that provide a set of physical quantities x_1, x_2, \dots, x_n . To obtain the result, calculations must be performed in accordance with the functional dependence (I.1).

$$F = f(x_1, x_2, \dots, x_n),$$

where F is the desired value of the indirect measurement, and x_1, x_2, \dots, x_n are the physical quantities measured directly using the devices. Thus, an indirect measurement is one whose result is calculated using the formula for F . Let us assume an experiment has been conducted, data (x, y, z) obtained from the devices and the errors of the direct measurements calculated. To estimate the error of an indirect measurement, the **error propagation formula** must be used. For any quantity that is a function of several variables (x, y, z) , the error in F is defined as

$$\Delta F = \sqrt{\left(\frac{\partial F}{\partial x}\right)^2 \Delta x^2 + \left(\frac{\partial F}{\partial y}\right)^2 \Delta y^2 + \left(\frac{\partial F}{\partial z}\right)^2 \Delta z^2}, \quad (\text{I.18})$$

where $\Delta x, \Delta y$ and Δz are the errors of direct measurement.

Let us consider a procedure for calculating the error of an indirect measurement using the volume of a parallelepiped as an example (formula I.18). Assume the measured dimensions are x, y, z with $x=L, y=H$ and $z=W$. The volume is $V=LHW$. To derive a formula for the error in V , let us compute the required partial derivatives and substitute them into formula (I.18). If unfamiliar with partial derivatives, perform the calculations as for ordinary derivatives, treating all variables as constant except the one with respect to which differentiation is taken.

$$\frac{\partial F}{\partial x} = \frac{\partial V}{\partial L} = HW, \quad \frac{\partial F}{\partial y} = \frac{\partial V}{\partial H} = LW, \quad \frac{\partial F}{\partial z} = \frac{\partial V}{\partial W} = LH.$$

Let us substitute the obtained expression into (I.18) and rewrite them in the form

$$\Delta F = \Delta V = LWH \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta W}{W}\right)^2 + \left(\frac{\Delta H}{H}\right)^2}. \quad (\text{I.19})$$

In this formula, ΔL , ΔW , ΔH are the absolute errors of direct measurements. Note that mean values should be substituted for L , W and H . The formula can be used to calculate the relative error ε , taking into account that $\varepsilon_V = \frac{\Delta V}{V}$,

$$\varepsilon_V = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta W}{W}\right)^2 + \left(\frac{\Delta H}{H}\right)^2}. \quad (\text{I.20})$$

Section 5

Least square method

Functional relationships between physical quantities

A physical experiment is aimed not only at measuring a specific quantity but also at identifying patterns between quantities and determining their functional relationship. For example, Ohm established that the current flowing through a conductor is directly proportional to the voltage across its ends and inversely proportional to its resistance. To verify Ohm's law and determine how the current I_i changes, it is necessary to obtain a set of current values corresponding to changes in voltage U_i and establish the functional dependence $I(U)$. The task of selecting a function that best describes the set of experimental data (I_i U_i) is addressed through regression analysis using the method of least squares. Let us assume a linear functional relationship of the form

$$y_i = a + bx_i, \quad (\text{I.21})$$

where y_i is the value of the variable Y_i predicted by equation (I.21). Since the straight line only approximately describes the set of experimental values, the predicted value y_i may not coincide with the experimental value Y_i . Deviations of predicted values from experimental values are called **regression residuals** and are denoted as $\Delta_i = Y_i - y_i$. The regression problem reduces to finding the regression coefficients a and b . The method for determining these coefficients is based on the least squares method, i.e. minimising the sum of the squares of the regression residuals. Solving this problem yields the values of coefficients a and b .

$$b = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2},$$
$$a = \bar{y} - b\bar{x}.$$

Let us consider the case of determining regression coefficients using the least squares method. Using the data in Table I.3, the coefficients can be calculated by the formulas:

$$b = \frac{\overline{xy} - \bar{x}\bar{y}}{x^2 - \bar{x}^2} = \frac{50,5 - 7,1 \times 5,5}{38,5 - (7,1)^2} = 1,3878,$$

$$a = \bar{y} - b\bar{x} = 7,1 - 1,3878 \times 5,5 = -0,533.$$

Thus, the result is $y_i = a + bx = -0.533 + 1.3878x$.

Table I.3

Experimental data set for finding functional dependence

N	y_i	x_i	x_i^2	$x_i y_i$
1	1	2	4	2
2	3	2	4	6
3	1	4	16	4
4	3	4	16	12
5	5	8	64	40
6	4	6	36	24
7	5	9	81	45
8	7	8	64	56
9	10	12	144	120
10	12	16	256	192
sum	51	71	685	501
mean	5.1	7.1	68.5	50.1

Let us compare the graph constructed from the coefficients obtained by the least squares method, $y_i = a + bx = -0.533 + 1.3878x$, with the graph based on the experimental data from Table I.3. Figure I.4 clearly shows that the straight line provides a good approximation of the experimental data.

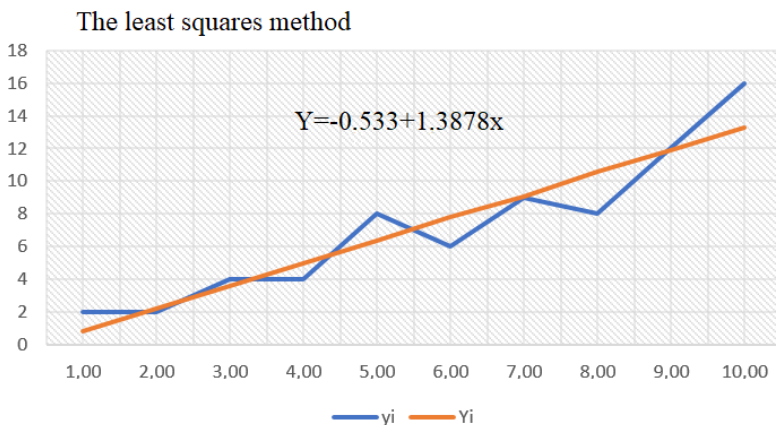


Fig. I.4. Graph of the curve based on experimental data (blue line) and on the coefficients obtained by the least squares method (orange line)

Conclusion

In each laboratory exercise, after performing the experiment and calculations and determining measurement errors, the student must draw a conclusion. The conclusion should reflect the results of the experimental study and report the outcomes achieved during the exercise. It should note whether the agreement between theory and experiment is confirmed, compare the obtained and standard values of physical quantities, analyse how the results depend on various parameters and indicate the student's level of mastery of the research method. If the experimental data diverge from theory, the phenomenon or law, or if the error in determining the desired value exceeds 10%, the possible reasons should be stated, or the experiment should be repeated.

Section 6
Dynamics: Theory
Laws of dynamics for translational and rotational motion

1. Frame of reference and coordinate system.
2. Newton's laws of translational motion. Forces in nature.
3. Newton's laws of rotational motion.

Frame of reference and coordinate system

Dynamics is a branch of physics that studies the causes of motion and the patterns of its change or absence. Classical dynamics is based on **Newton's laws**, the laws of conservation of momentum and the laws of conservation of energy. Its fundamental concepts include **mass**, **force**, **work** and **energy**. The laws of dynamics are applicable only within a certain **frame of reference**. A reference system consists of a coordinate system specifying a reference point and time, which can be chosen arbitrarily depending on the task.

The laws of dynamics are formulated for material points. A **material point** is a physical abstraction of a body whose dimensions can be neglected compared with the distances to the body in the chosen coordinate system. For example, bodies on the Earth's surface can be treated as material points, since their sizes are negligible compared with the Earth's radius, $R=6371 \text{ km}$.

Newton's laws of translational motion. Forces in nature

Newton's first law states that any material point, or body, remains at rest or moves with constant velocity in a straight line until acted upon by other bodies. Put differently, an object at rest stays at rest and an object in motion keeps moving at constant velocity unless acted upon by a net external force. The First Law also permits defining force as that which produces a change in an object's

motion. The tendency of an object to resist any change in its velocity is called **inertia**. The measure of a body's inertia is its mass, referred to as **inertial mass**.

Mass is a property of an object that quantifies its resistance to a change in velocity. It is a scalar quantity. The SI unit of mass is the kilogram (kg).

Newton's First Law does not hold in every frame of reference. Frames in which it is valid are called **inertial frames of reference**, where a material point free from external influences is either at rest or moves with constant velocity.

Newton's second law

The second law states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

$$\vec{a} = \frac{\vec{F}}{m}. \quad (\text{D.1})$$

A force causes a change in motion measured by acceleration. Net force \vec{F} is the vector sum of all forces acting on an object F_i .

$$\vec{F} = \sum_i^N \vec{F}_i. \quad (\text{D.2})$$

Acceleration is a vector and has the same direction as the net force. The SI unit of force is the newton (N), where $1 \text{ N} = 1 \text{ kg}\cdot\text{m}/\text{s}^2$.

Newton's second law can be expressed in terms of components within a chosen frame of reference. For example, the projection of the net force on the X-axis equals the mass multiplied by the acceleration component along the X-axis: $a_x m = F_x$.

Momentum of a body. Equation of motion

A physical quantity equal to the product of mass m and velocity v is called **momentum**.

$$\vec{p} = m\vec{v}. \quad (\text{D.3})$$

Thus, Newton's second law (D.1) can be expressed in a different form since acceleration represents the change in velocity per unit time. The **equation of motion** then takes the form:

$$\vec{F} = \frac{d\vec{p}}{dt}. \quad (\text{D.4})$$

Equation (D.4) shows that the rate of change of a body's momentum depends on the net force applied to this body. Therefore, equation (D.4) can be rewritten in another form:

$$\vec{F}t = m\vec{v}. \quad (\text{D.5})$$

$\vec{F}t$ stands for **force impulse**. The equation (D.5) shows that impulse is equal to the change in momentum. If no forces act on a body, its momentum remains constant. Thus, in the absence of external forces (D.4), the **total momentum is conserved**, which is one of the fundamental **conservation laws**.

It should be noted that by force we mean the net force, since multiple forces typically act on a body. Newton's law can be extended to a system of several bodies. For an isolated system, momentum refers to the total momentum of the system. When the net external force is zero $\vec{F} = 0$, $\vec{p} = 0$, Newton's law implies the **law of conservation of total momentum** in a system.

Newton's third law

When two bodies interact, the force exerted by body 1 on body 2 is equal in magnitude and opposite in direction to the force exerted by body 2 on body 1.

$$\vec{F}_{12} = -\vec{F}_{21}.$$

One of these forces is called the action force, and the other is the reaction force. Let us consider examples of different forces acting on a body.

Forces in nature.

Gravitational forces act between masses. The law of universal gravitation states that the gravitational force is proportional to the

product of the masses of the interacting bodies and inversely proportional to the square of the distance between them. In vector form, the law can be written as:

$$\vec{F}_{12} = -G \frac{m_1 m_2 \vec{r}_{12}}{r^2}, \quad (\text{D.6})$$

where $\frac{\vec{r}_{12}}{r}$ is the unit radius vector drawn from body 1 to body 2.

This law (D.6) is valid for point masses (material points), that is, bodies whose dimensions are much smaller than the distance between them. \vec{r}_{12} is the radius vector drawn from body 1 to body 2. Note that the law of universal gravitation also applies to spherical bodies (Fig. D.1).

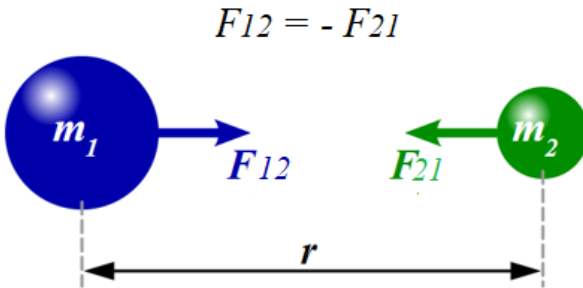


Fig. D.1. Spherical bodies with masses m_1 and m_2 subject to the force of attraction — the force of universal gravitation

In this case, the distance between the centres of the spheres is taken as the distance between the bodies. The law of universal gravitation reveals another property of mass: **mass is a measure of gravity**. It has been experimentally established that **gravitational mass is equal to inertial mass**. Consider an example — calculating the acceleration with which a body moves under the action of Earth's gravity. This force is directed towards the centre of the Earth. Applying Newton's second law, we can write:

$$F = mg.$$

The equation is written in scalar form, as the direction of acceleration coincides with that of the force. If no other forces act on the body, the body will fall with the acceleration due to gravity. According to the law of universal gravitation, we obtain:

$$G \frac{m M_{Earth}}{R^2} = mg, \quad G \frac{M_{Earth}}{R^2} = g,$$

where R is the distance from the centre of the Earth to an object on its surface. Therefore, R is the radius of the Earth. As a result, gravitational acceleration is the same for all bodies on Earth, and it does not depend on the mass of the body. The force that attracts bodies to the Earth, equal to mg , is called the **gravitational force**.

Substituting the numerical values,

$$G = 6.67430 (15) \cdot 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2), M = 5.972 \cdot 10^{24} \text{ kg}$$

$$\text{and } R = 6.371 \cdot 10^6 \text{ yields } g = 9.8 \text{ m/s}^2.$$

The acceleration of gravity, however, will vary from planet to planet and depend on the position of the body relative to the planet's surface. Formula (D.6) can be universally applied to calculate gravity on any planet.

If a body is located on the surface of a planet and is in a state of equilibrium, two forces act on the body: gravity and the normal reaction force. The net force, which is the vector sum of these forces, is zero.

Figure D.2 shows gravity mg , where **mg is the action force**, and **N , the normal reaction force**. When the body is at rest or moving with constant velocity, $mg = N$. **The force equal in magnitude to the force with which a body acts on other bodies is called the weight** of the body, while the mass of a body is an intrinsic property and does not change. Therefore, the gravitational force can only vary with changes in gravitational acceleration, whereas the weight depends on the body's motion and can change.

Let us solve a problem. A body is suspended by an inextensible thread inside a lift. Determine the weight of the body in three situa-

tions: 1) the lift moves uniformly with velocity v ; 2) the lift moves downward with acceleration a ; 3) the lift moves upward with acceleration a (Fig. D.3, a , b , c).

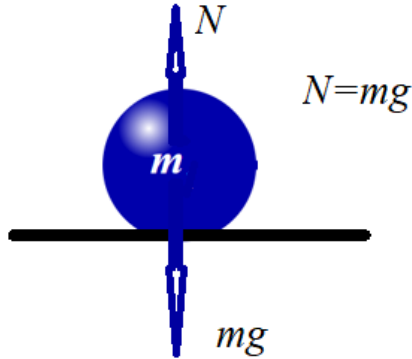


Fig. D.2. Gravity mg and normal reaction force N acting on a body in equilibrium. The net force is zero

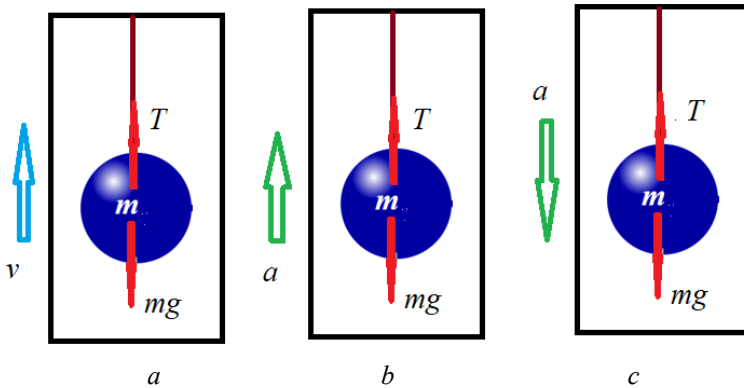


Fig. D.3. Three scenarios for a body suspended by a thread in a lift:
 a — the lift moves at a constant velocity v ;
 b — the lift accelerates downwards with acceleration a ;
 c — the lift accelerates upwards with acceleration a

Let us write Newton's law for the three scenarios, choosing a vertical axis directed upwards as the reference system:

a) $T - mg = 0$. The weight is numerically equal to the tension in the thread; in this case, the weight equals the gravitational force;

b) $T - mg = ma, T = mg + ma$. In this case, the weight increases by ma ;

c) $T - mg = -ma, T = mg - ma$. The weight decreases by ma . If the acceleration equals the acceleration of gravity, weightlessness occurs. Astronauts experience this when a spacecraft travels in a circular orbit around the Earth.

Friction forces arise during motion or when there is the potential for relative movement between bodies in contact. The maximum friction force F is equal to the product of the coefficient of friction and the normal force, which is numerically equal to the reaction force:

$$F = \mu N, \quad (\text{D.7})$$

where N is the normal force and μ is the coefficient of friction between the two surfaces. The value of μ depends on the materials in contact. The **friction force** is always directed opposite to the velocity of the body.

Elastic force.

When an elastic body, such as a spring or thread, is stretched, an elastic force arises, tending to return it to its equilibrium position (Fig. D.4). The elastic force is proportional to the displacement from the equilibrium position and is always directed towards it.

$$\vec{F} = -k \cdot \vec{r}, \quad (\text{D.8})$$

where k is the elasticity coefficient or the spring stiffness coefficient, and \vec{r} is the displacement of the body from the equilibrium position. Note the minus sign in (D.8), which indicates that the force is always directed opposite to the displacement.

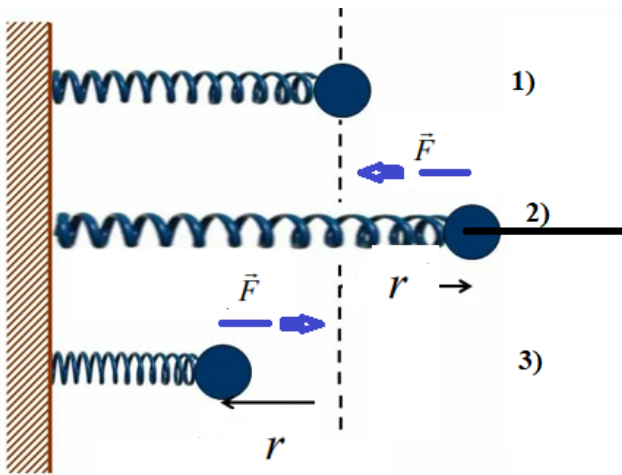


Fig. D.4. The action of an elastic force:
 1 — at the equilibrium position of the elastic body ($F=0$);
 2 — when the body is stretched; 3 — when it is compressed

Newton's laws for rotational motion

Newton's second law for translational motion explains the cause of a change in a body's velocity. It states that a force produces acceleration. This raises the question: can the law be applied to rotational motion?

Does the action of a force always cause a change in the rotational velocity of a body? Experience shows that a force whose line of action passes through the axis of rotation does not change the rotational velocity, making rotation of the body impossible. Torque is crucial in this context. The effect of a force depends on its point of application relative to the axis of rotation. **Torque is calculated as the vector product of the radius vector $\vec{r}(t) = R$** , which specifies the position of the material point relative to the axis, and the force \vec{F} acting on the point (Fig. D.5).

$$\vec{\tau} = [\overrightarrow{r(t)} \cdot \vec{F}]. \quad (\text{D.9})$$

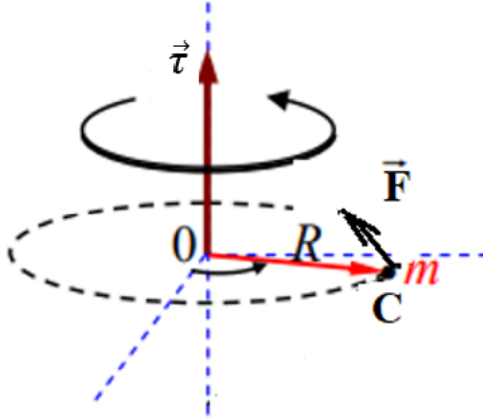


Fig. D.5. The direction of torque is determined by the right-hand rule for vectors: $\overrightarrow{r(t)}$, \vec{F} , $\vec{\tau}$

In translational motion, the extent to which a force accelerates a body depends on its inertia, defined by its mass. For rotational motion, the measure of inertia is the **moment of inertia**. The **moment of inertia of a material point** is a physical quantity equal to the product of the mass of the body and the square of the distance to the axis of rotation:

$$J = mR^2. \quad (\text{D.10})$$

To describe rotational motion, both linear and angular characteristics are used. When a material point of mass m moves around an axis O , it rotates through an angle $\Delta\varphi$ over a time interval Δt . During this time, the point travels a distance dl along the circle. The direction of the angular displacement is defined as a vector perpendicular to the plane of rotation of the mass m . Its orientation is determined using the right-hand rule (Fig. D.6, *a*).

$$dl = \Delta\varphi \cdot r(t),$$

where $r(t)$ is the radius vector, which specifies the position of the material point (Fig. D.6, *b*). Thus, the angular velocity of mass m , by definition, is equal to $\vec{\omega} = \frac{d\varphi}{dt}$, and it is directed perpendicular to the plane of rotation. Linear velocity at point C is always directed tangentially to the circle (Fig. D.6, *c*). If, at a given moment, the vector $\vec{r}(t) = \vec{R}$, the relationship between linear and angular velocities $\vec{V} = [\vec{\omega}, \vec{R}]$ can be established. The linear velocity vector, the angular velocity vector and the radius vector together form a right-handed triad.

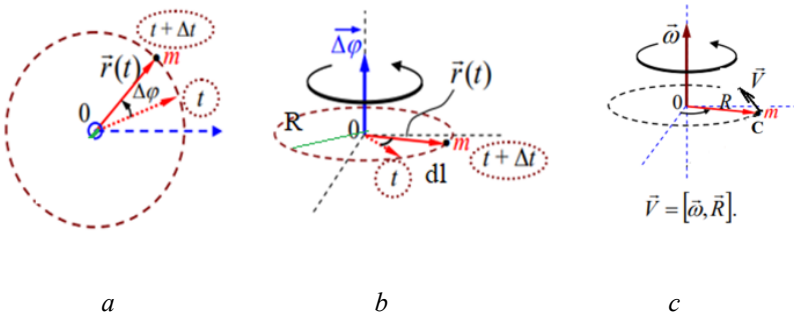


Fig. D.6. Notation of the main kinematic characteristics of a rotating body: *a* — the angular displacement of a point $\Delta\varphi$; *b* — the angular displacement vector $\Delta\varphi$, the radius vector and the linear displacement dl ; *c* — the direction of angular velocity $\vec{\omega}$ and linear velocity \vec{V} at a given point in time $\vec{r}(t) = \vec{R}$

The equation describing the rotational dynamics of a material point of mass m , analogous to Newton's second law, is given by:

$$\vec{M} = I\vec{\varepsilon}, \quad (\text{D.11}),$$

where $\vec{\varepsilon} = \frac{d\vec{\omega}}{dt}$ is the angular acceleration.

Summary of basic formulas (Table D.1)

Table D.1

Comparative characteristics for describing translational and rotational motion

Motion characteristics	Translational movement	Rotational movement	Review of basic formulas. Relationship between characteristics
Inertia	Mass, m , [kg]	Moment of inertia, I , [kg · m ²]	$J = mR^2$
Change in the position of a material point	Path, s or l [m] Displacement, $\vec{r}(t)$	Angle of rotation, φ [rad]	$dl = \Delta\varphi \cdot r(t)$
Rate of body position change	Speed (scalar quantity), $v = \frac{ds}{dt}$. Velocity (vector quantity), $\vec{V} = \frac{d\vec{r}}{dt}$, [m/s]	Angular velocity, $\vec{\omega} = \frac{d\varphi}{dt}$, [rag/s]	$\vec{V} = [\vec{\omega} \times \vec{R}]$
Changes in body velocity	Acceleration, $\vec{a} = \frac{d\vec{v}}{dt}$, [m/s ²]	Angular acceleration, $\vec{\varepsilon} = \frac{d\vec{\omega}}{dt}$, [rag/s ²]	$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d(\vec{n})v}{dt} = \frac{d(\vec{n})}{dt}v + \frac{dv}{dt}\vec{n}$ \vec{n} -unit velocity vector $\vec{a} = \vec{a}_r + \vec{a}_n$
The effect of other bodies on a body, causing a change in its velocity	Force, \vec{F} , [N] = [kg · m/s ²]	Momentum of force, \vec{M} , [kg · m ² /s ²]	$\vec{M} = [\vec{r}(t) \cdot \vec{F}]$
Newton's second law	$\vec{F} = m\vec{a}$	$\vec{M} = I\vec{\varepsilon}$	

Uniform translational motion along the X-axis.

For uniform motion along the X-axis, the speed remains constant, and the path taken equals numerically the magnitude of the displacement.

$$S = x(t) = v_{0x}t.$$

$$v(t) = v_{0x}.$$

Figure (D.7) shows graphs of velocity and distance as functions of time.

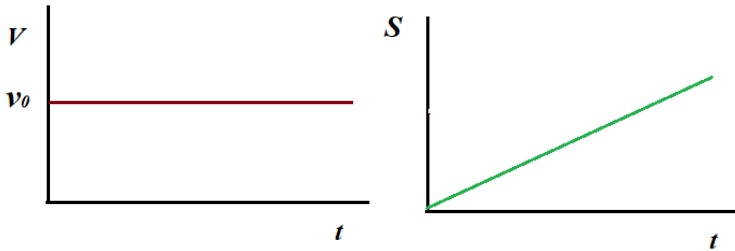


Fig. D.7. Graphs of speed $v(t)$ and path $S(t)$ versus time

Uniform movement around a circle.

$$S = 2\pi R = vT, \quad \omega = 2\pi\nu, \quad \omega = \frac{2\pi}{T},$$

where the path S equals the length of a circle with radius R ; v is the linear velocity of the body; T is the rotation period, ω is the angular velocity in rad/s, and ν is the rotation frequency in Hz (1/s).

Equally alternating translational motion.

A body's motion is uniformly variable if its acceleration does not depend on time. In this case, speed and displacement are described by equations of the form:

$$X(t) = v_{0x}t + \frac{a_x t^2}{2},$$

$$v(t) = v_{0x} + a_x t.$$

Problems

1. A material point is attached to a wall by a spring and can move along the horizontal axis X . An elastic force acts on the material point, proportional to the displacement X of the spring. The spring elasticity coefficient is k . X_0 is the coordinate of the material point in the absence of spring deformation. Write down Newton's law for the material point and the force acting on it, then calculate the force if $X=20$ cm, $X_0=10$ cm, $k=0.2$ N/m. Draw a graph of the elastic force versus displacement.

2. A body rolls down an inclined plane at an angle $\varphi=45^\circ$. Identify the forces acting on the body. Write down Newton's law for the body and calculate the acceleration of the body if the coefficient of friction is $k=0.02$.

3. A body rolls down a chute. Determine the height from which the body must be released so that it completes a vertical loop of radius R . The body must not fall at the highest point A . Use Newton's law.

4. A body moves around the Earth. Determine the speed the body must have in order not to fall to the Earth. Identify the forces acting on the body. Write down Newton's law and calculate the centripetal acceleration of the body. The radius of the Earth is $R=6400$ km. Perform the calculation in the SI system.

5. A body falls from a balloon at height $H=50$ m above the ground. Determine the speed of the body when it reaches the Earth's surface. Identify the forces acting on the body and write down Newton's law for this motion.

6. The human body can be modelled as a homogeneous cylinder with radius $r=30$ cm, height $h=1.7$ m, and mass $m=70$ kg. Determine the moment of inertia of the human body in a standing and lying position relative to a vertical axis passing through the centre of the cylinder, approximately corresponding to the centre of mass of the human body.

7. A stone weighing 0.2 kg is thrown horizontally from a tower 40 m high. Neglecting air resistance, determine the speed at which

the stone will strike the ground, the time it takes to reach the ground and the distance from the base of the tower to the point where the stone will land.

8. Apply Newton's second law to the motion of the weights in an Atwood machine. Determine the acceleration of the system and the tension in the thread given that the masses of the weights are $m_1=50\text{ g}$ and $m_2=100\text{ g}$; the disk has mass $M=500\text{ g}$ and radius $R=60\text{ cm}$.

Guidelines for solving dynamics problems.

As an example, let us consider a problem in dynamics and its solution. In dynamics, the main role is played not only by forces, moments of forces and impulses, but also by their directions and related characteristics, since all these quantities are vectors. Therefore, when solving problems, particular attention must be paid to choosing a reference system, relative to which the motion is analysed. It is also crucial to determine the directions of forces and torques. Remember that Newton's laws are valid only in inertial reference systems.

Problem. A cylinder rolls down an inclined plane. Find the acceleration of the centre of its mass and its final velocity. Assume that the cylinder rolls down without slipping (see Fig. D.8).

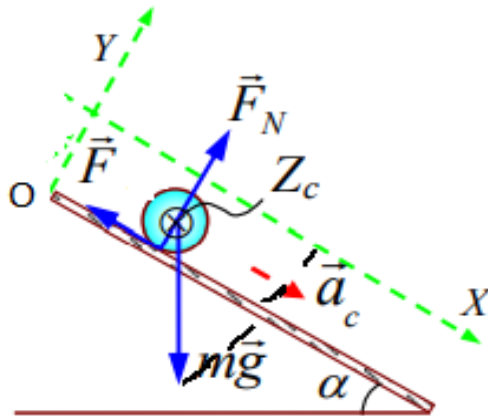


Fig. D.8. A cylinder rolls down an inclined plane

Let us consider the forces acting on a body and their directions.

1. The force of gravity, acting downwards.

2. The body exerts a force on the plane, and according to Newton's third law, the plane exerts a reaction force on the body, equal in magnitude and directed along the normal to the surface.

3. A friction force acts between the body and the surface, directed opposite to the motion (against the velocity).

Choose a coordinate system to simplify the application of Newton's law. Let one axis lie along the surface of the board (OX) and the other perpendicular to it (OY) (Fig. D.8). This coordinate system is convenient because the sum of forces along the Y-axis is zero, and the body moves only along the OX axis.

Write Newton's second law in coordinate form. To find the projections of all forces along the OX axis, let us drop a perpendicular from the tip of the gravity vector mg to the x-axis. This gives the equation

$$ma_c = mg \cdot \sin\alpha - F_{fr}.$$

4. Similarly, let us write Newton's equation in projections onto the Y-axis. Since the body does not move along the Y-axis, its acceleration is zero.

$$0 = mg \cdot \cos\alpha - F_N.$$

5. Let us write the equation of motion for a rotating body. Angular acceleration is produced by a force acting along the rim of the cylinder. The moments of gravity and the normal force are zero, since their lines of action pass along the axis of rotation and their arm is equal to zero.

$$I_c\beta = F \cdot R,$$

where I_c is the moment of inertia.

$$I_c = \frac{mR^2}{2}.$$

6. Let us consider the relationship between angular and linear acceleration.

$$a_c = \beta \cdot R.$$

Combining the above equations, we obtain an expression for the friction force. Combining the above expressions, we obtain:

$$a_c = \frac{mg \sin \alpha}{m + \frac{I_c}{R^2}} = \frac{2}{3} g \sin \alpha.$$

Sliding without friction occurs with acceleration:

$$g \sin \alpha.$$

The coefficient $\frac{2}{3}$ that appears in our result is the ‘price’ of the body’s rotational motion during rolling. This ‘price’ depends on the moment of inertia of the body, that is, on the distribution of its mass relative to the Z_c axis. For example, a solid cylinder rolls

$$\left(I_c = \frac{mR^2}{2} \right)$$

faster than a hollow one

$$I_c = mR^2,$$

regardless of mass and radius.

Moreover, linear velocity of the centre of mass and angular rolling speed grows linearly with time regardless of mass and radius.

$$v_c = \left(\frac{2}{3} g \sin \alpha \right) \cdot t; \quad \omega = \left(\frac{2}{3} \frac{g}{R} \sin \alpha \right) \cdot t.$$

Section 7
Energy. Work. Conservation laws.
Elastic and inelastic collisions. Theory

The **work** W done by a constant force when moving a body from point A to point B is determined by the scalar product \vec{F} of the force and the displacement $\vec{\Delta r}$. Numerically, the work equals the product of the force, the displacement AB and $\cos \theta$, where θ is the angle between the force and displacement vectors.

$$W = \vec{F} \vec{\Delta r} = Fr \cos. \quad (\text{E.1})$$

Energy can be classified as **kinetic energy**, **potential energy** and **internal energy**.

Kinetic energy is associated with the motion of a system's elements and depends on the speed of the body. The kinetic energy of a material point is given by:

$$K = \frac{mv^2}{2}. \quad (\text{E.2})$$

Obtaining the kinetic energy of the entire system requires calculating the sum of the kinetic energies of all N material points m_i comprising the system.

$$K = \sum_i^N \frac{m_i v_i^2}{2}.$$

When dealing with a rigid body, all points of which move with the same linear velocity, one must consider not only the kinetic energy of translational motion but also the kinetic energy of rotational motion if the body rotates about a certain axis. When the body rotates, the kinetic energy is the sum of the kinetic energies of all individual points N. The kinetic energy can be expressed as:

$$K = \sum_i^N \frac{m_i v_i^2}{2} = \sum_i^N \frac{m_i \omega^2 r_i^2}{2} = \frac{\omega^2}{2} \sum_i^N m_i r_i^2 = \frac{I \omega^2}{2}, \quad (\text{E.3})$$

where the **relationship between linear and angular velocities** is taken into account and is given by $\mathbf{v} = \boldsymbol{\omega} \mathbf{r}$ (Table D.1, Fig. D.6, c). In

$\vec{V} = [\vec{\omega} \times \vec{R}]$, ω is the angular velocity, r is the radius vector from axis of rotation to material point.

I stands for the moment of inertia of a rotating body

$$I = \sum_i^N m_i r_i^2.$$

Note that the angular velocity ω of all points of the body is the same and is directed perpendicular to the plane of the body.

Potential energy is determined by the configuration of the system. A specific expression for potential energy can be obtained from the fact that the work done against conservative forces is stored as potential energy. As an example, let us find an expression for gravitational and elastic potential energy.

1. Consider an object located on the surface of the Earth. To raise it to a height H , it is necessary to overcome the force of gravity $F=mg$. Let us determine the work W required to move a body of mass m to a height H against the force of gravity. Gravity is a conservative force, as its work does not depend on the path taken by the body. Therefore, the work equals the force of gravity multiplied by the displacement of the body between the points:

$$W = mgH = U. \quad (\text{E.4})$$

In this case, work is equal to the gravitational potential energy.

2. A body of mass m is attached to a spring (Fig. E.1). In the equilibrium position, a body of mass m is located at a distance r_0 from the origin O. When the spring is stretched, an elastic force acts on the body according to Hooke's law $\vec{F} = -k\vec{r}$ (Fig. E.1, *a*). Let us determine the work required to stretch the spring by a distance x (Fig. E.1, *b*), using the general definition of work and considering that the elastic force F depends on the distance $F = kx$:

$$W = \int_0^x kx dx = \frac{kx^2}{2} = U. \quad (\text{E.5})$$

Thus, the **potential energy of an elastic spring** depends on the spring stiffness coefficient and is proportional to the square of the spring's displacement from the equilibrium position x . **Another**

way to determine the work done by elastic forces, or the energy stored when compressing a spring, is to use a graph of force versus spring displacement (Fig. E.1, *a*). The area of the figure enclosed by the force graph and the abscissa axis (x-axis) will be numerically equal to the spring energy.

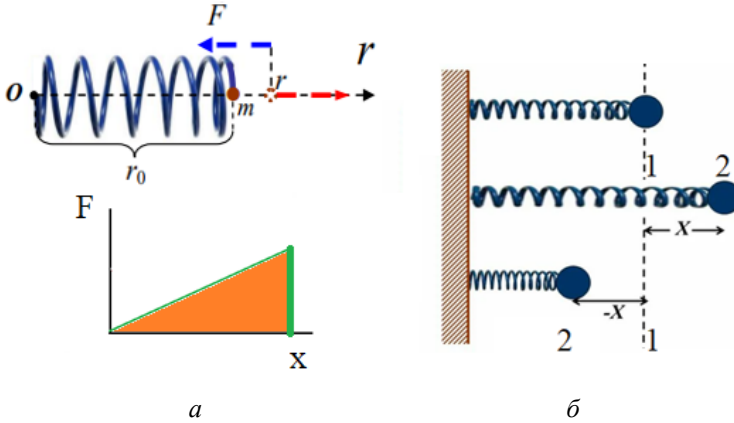


Fig. E.1. A spring under an elastic force obeying Hooke's law

$$\vec{F} = -k\vec{r}$$

Internal energy is the sum of the kinetic and potential energy of all elements of a body, while **mechanical energy** is the sum of kinetic and potential energy. **Internal energy** depends on the body's temperature.

Law of conservation of energy.

The law of conservation of energy states that the total energy of an isolated system is conserved. The total energy of a system is the sum of kinetic, potential and internal energy:

$$K + U + T = \text{constant},$$

where T is the internal energy of the body, which depends on its temperature. In the special case of an isolated system where the in-

ternal energy does not change, the law of conservation of mechanical energy applies. In non-isolated systems, energy can be transferred to other bodies, so the law of conservation of energy does not hold. It is therefore important to ensure that the chosen system is isolated.

The moment of inertia of a rotating body. Huygens — Steiner theorem.

The moment of inertia of a body depends on the position of the axis around which the body rotates (Fig. E.2). The moment of inertia relative to an arbitrary axis can be determined using the **Huygens — Steiner theorem**, which states that the moment of inertia I_0 of a rigid body about an arbitrary axis is equal to the sum of the moment of inertia I_c about a parallel axis passing through the centre of mass C and the product of the body's mass and the square of the distance r between the axes:

$$I_0 = I_c + mr^2. \quad (\text{E.6})$$

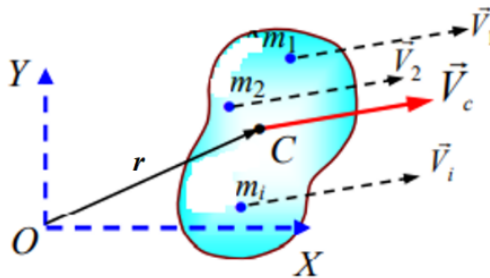


Fig. E.2. A rigid body rotating about the axis OY

Let us calculate the moment of inertia of a rod relative to the axis of rotation passing through its end. To do this, we use the formula for the moment of inertia of the rod relative to the axis through its centre of gravity, referring to Figure E.3, *b*.

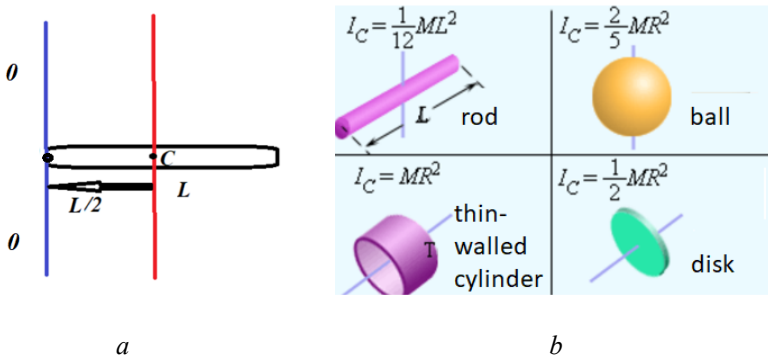


Fig. E.3. The moment of inertia of a rod of length L relative to an axis passing through its end $O-O$ (a), and the moment of inertia of selected bodies relative to an axis passing through the centre of gravity (b)

The moment of inertia of the rod of length L relative to an axis passing through its end OO according to the Huygens-Stern theorem is equal to

$$I_0 = I_c + mr^2 = \frac{1}{12}mL^2 + \frac{1}{4}mL^2 = \frac{1}{3}mL^2. \quad (\text{E.7})$$

The definition of *moment of inertia* can be used to determine the moment of inertia of a rod of length L and cross-sectional area A about an axis perpendicular to the rod through one end $O-O$. Suppose the density of the rod is ρ ; the volume of an element of length dx is given by $A dx$, and the mass dm of the element is $\rho A dx$. The moment of inertia of this element, located at a distance x from the axis of rotation, is $dI = \rho A x^2 dx$, and the moment of inertia of the entire rod is obtained by summing, or integrating, the contributions of each element of the rod. Therefore,

$$I = \int_0^L dI = \int_0^L \rho A x^2 dx = \left[\rho A \frac{x^3}{3} \right]_0^L = \rho A \frac{L^3}{3}.$$

Since $V=AL$ is the volume of the rod, the mass of the rod m is given by ρAL . Let us write down the moment of inertia of the rod relative to the perpendicular axis passing through one of its ends.

$$I = \frac{1}{3}mL^2.$$

Elastic and inelastic collisions.

Let us consider different types of collisions of bodies in an isolated system. Collisions are classified as elastic or perfectly elastic, and as inelastic or perfectly inelastic. **Absolutely elastic collisions** are collisions in which the internal energy of the colliding bodies does not change and the law of conservation of mechanical energy is satisfied. An **absolutely inelastic collision** is an impact in which the bodies come together and merge. Here, the law of conservation of mechanical energy is violated due to the transition of kinetic energy into work to overcome the friction force. In both cases, however, the law of conservation of momentum is satisfied.

Angular momentum.

If bodies in a closed system undergo rotational motion about a certain axis, this motion is characterised by angular momentum. Angular momentum \vec{L} is a vector quantity defined by the vector product of the position vector of a particle \vec{r} , relative to the axis of rotation, and its linear momentum \vec{p} :

$$\vec{L} = [\vec{r} \times \vec{p}]. \quad (\text{E.8})$$

The magnitude of the angular momentum is $L = r mv \sin\varphi$.

If a body rotates in a circle of radius r , a useful expression for its angular momentum can be obtained:

$$L = mr^2\omega = I\omega. \quad (\text{E.9})$$

It follows that the change in momentum over time is equal to the torque.

$$\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \vec{F} = \vec{M}. \quad (\text{E.10})$$

This expression is the fundamental equation of rotational motion. Let us consider an example of a particle moving under the ac-

tion of so-called central forces, whose line of action passes through a single point (the centre of force) and depends only on the distance to this point. The general expression for a central force is:

$$\vec{F} = \frac{\vec{r}}{r} F(r).$$

Examples of **central forces** include gravitational force, Coulomb force and elastic force. The distinctive feature of these forces is that the work done by a central force does not depend on the path, but only on the initial and final positions of the body. In addition, the torque of a central force is zero:

$$\vec{\tau} = [\vec{r} \times \frac{\vec{r}}{r} F(r)].$$

Consequently, the **angular momentum is conserved**, i. e. both its magnitude,

$$L = mr^2\omega = \text{constant},$$

and its direction perpendicular to the plane of motion are conserved. From this, an important conclusion can be drawn: the plane of a body's orbital motion does not change, so, for example, spiral motion is impossible. It can be shown that equation (E.10) is valid for a system of bodies. Here, \vec{L} denotes the sum of the angular momentum of all bodies and \vec{M} the net external torque. If the net external torque \vec{M} acting on the system equals zero, the net angular momentum is conserved:

$$\frac{d\vec{L}}{dt} = 0. \quad (\text{E.11})$$

An example of an isolated system is a rotating platform (friction in the bearings neglected) with a person standing on it. In this case, the total angular momentum, equal to the sum of the angular momenta of the person and the platform, remains constant. Suppose the platform with the person standing at its centre rotates with an angular velocity ω_1 . How will the rotation speed of the platform, ω_2 , change if the person spreads their arms? Let the moment of inertia of the platform be I_1 and the moment of inertia of the

person with arms spread be I_2 . Applying the law of conservation of angular momentum, we can write the following relationship:

$$L_1 = L_2, I_1 \omega_1 = (I_1 + I_2) \omega_2.$$

Let us assume that at the initial moment, the moment of inertia of the person standing at the centre of the platform is zero. Thus, from the equation

$$\omega_2 = \frac{I_1 \omega_1}{(I_1 + I_2)}$$

it follows that the angular velocity will decrease.

Benjamin Robins' method for measuring bullet velocity.

Benjamin Robins, an English mathematician and military engineer, was the first to invent a device and a practical method for measuring the velocity of bullets and projectiles. He described this in his work *New Principles of Gunnery*, first published in 1742. Improved through the development of science and technology, this method remains relevant for measuring bullet velocity. Robins used a physical pendulum with a trap into which a bullet flew and became embedded. Let us now consider the physical processes that occur in this setup after the bullet is fired (Fig. E.4).

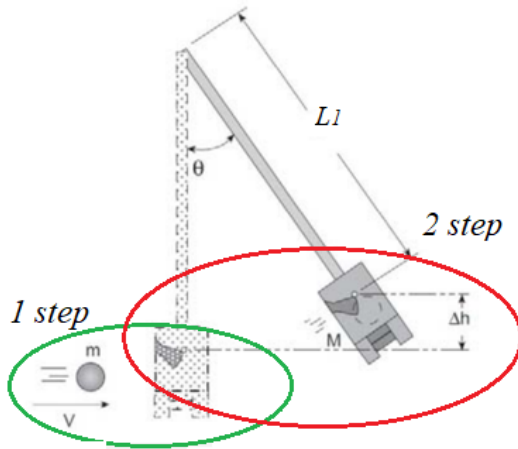


Fig. E.4. A physical pendulum with a trap for measuring bullet speed

After being fired, the bullet falls into a trap fixed in a stationary pendulum. In this case, the bullet undergoes an absolutely inelastic collision with the pendulum, which captures the bullet and absorbs its energy. The law of conservation of mechanical energy is not satisfied (step 1). If the duration of the bullet's impact on the pendulum is small compared to the oscillation period T of the physical pendulum, the pendulum does not have time to deviate significantly from its initial position during the impact. Furthermore, since the moment of external forces is small (external forces are much weaker than internal ones), the bullet-pendulum system can be treated as quasi-closed, and the law of conservation of angular momentum (E.11) can be applied.

$$m_1 v L_1 = I \omega, \quad (\text{E.12})$$

where m_1 is the mass of the bullet, v is the speed of the bullet, L_1 is the distance from the axis of the pendulum to the point where the bullet hits it, and I is the moment of inertia of the pendulum with the bullet relative to the axis of rotation of the physical pendulum. In this case, the momentum of inertia is given by:

$$I = (m_1 + m_3)L_1^2 + \frac{m_2 L_2^2}{3}, \quad (\text{E.13})$$

where m_1 is the mass of the bullet, m_2 is the mass of the rod, m_3 is the mass of the trap, and L_2 is the length of the rod.

After the impact, the pendulum with the embedded bullet moves at a certain speed and swings through an angle α . The kinetic energy is converted into the potential energy of the pendulum with the bullet; friction is negligible. Air resistance can also be ignored, as the speeds are relatively low.

At this stage (step 2), the law of conservation of mechanical energy applies. As the pendulum rises, its centre of mass ascends to a height h . In this case, the law of conservation of mechanical energy can be expressed as:

$$\frac{1}{2} \omega^2 = (m_1 + m_2 + m_3)gh,$$

where $h = R_{c.m.}(1 - \cos\alpha) = 2R_{c.m.}\sin^2\left(\frac{\alpha}{2}\right)$, $R_{c.m.}$ is the distance between the centre of mass and the suspension point.

II. LABORATORY

Laboratory 1 Determining the volume of a parallelepiped

The objectives of the lab are to determine the volume V of the parallelepiped and to calculate the measurement errors of its sides and volume.

Instruments and accessories:

1. Parallelepiped.
2. Caliper.

Theoretical framework. Determining the volume of a parallelepiped using a caliper. Calculating the error of indirect measurements.

The volume of the parallelepiped is $V=LHW$, where L , H , and W are the dimensions of the parallelepiped's dimensions: length L , height H and width W , respectively.

In this experiment, to determine the volume of a parallelepiped, we first measure its length L , height H and width W , and then calculate the volume using the formula $V=LHW$.

To measure the dimensions of a parallelepiped, a caliper or micrometer can be used. A micrometer provides higher accuracy, but the parallelepiped's length may exceed the micrometer's maximum range. A caliper is the most suitable device for measuring the dimensions of a parallelepiped. Measuring dimensions is a direct measurement, as the object's size is compared with a standard. Calculating the volume of the parallelepiped is an indirect measurement, since it requires using a formula.

It should be noted that the procedures for calculating the errors of direct and indirect measurements differ. The method for these calculations is described in the **Introduction**.

Let us consider the **principle of operation of a caliper** and how to use it to determine the size of an object (Fig. 1.1). Vernier calipers are employed to measure the height, diameter and depth of small objects. Their operation is based on the use of two scales and the ‘difference between two scales’ principle, which increases measurement accuracy. The main parts of a vernier caliper are the main scale, vernier scale, measuring jaws, locking screw and depth gauge (Fig. 1.1). The accuracy of a caliper is usually indicated on the scale; in Figure 1.1, the smallest reading is 0.05 mm.

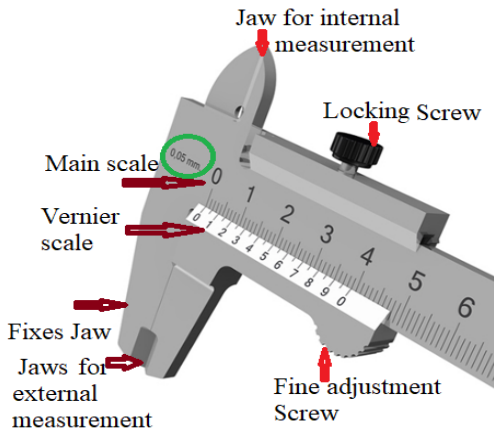


Fig. 1.1. Vernier calipers

First, we take the readings on the main scale, where one scale division equals 1 mm. In our case, the reading is approximately 6 mm. To obtain the exact value, we use the vernier scale. The division value on the vernier scale is usually indicated; in our case, it is 0.05 mm. Next, we identify the division on the vernier scale that exactly aligns with a division on the main scale. Figure 1.2 shows

examples of readings on a vernier caliper with a vernier division value of 0.05 mm. For instance, one reading is 6 mm + 0.45 mm = 6.45 mm, and another is 1 mm + 0.65 mm = 1.65 mm.

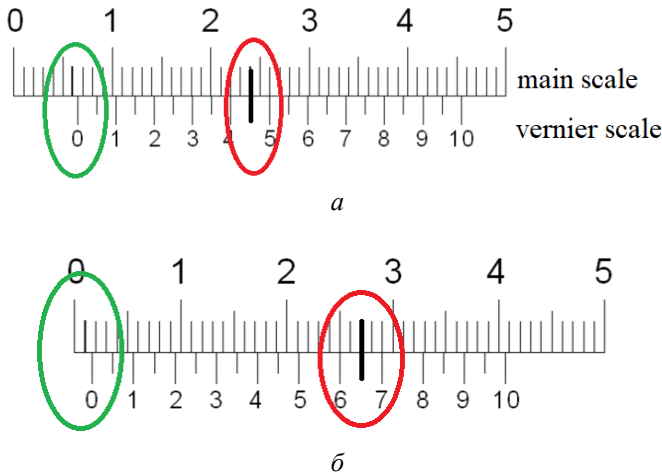


Fig. 1.2. The main and vernier scales of a caliper.

Two examples of readings on a caliper:

- a* — the vernier scale reading of 45 exactly aligns with a main scale division, giving a measurement of **6 mm + 0.45 mm = 6.45 mm**;
- b* — the vernier scale reading of 65 aligns with a main scale division, giving a measurement of **1 mm + 0.65 mm = 1.65 mm**

For example, two different readings are given. The first is 6 mm + 0.45 mm = 6.45 mm, the second instrument reading is 1 mm + 0.65 mm = 1.65 mm.

Procedure.

The procedure for the experiment is as follows:

1. Before measuring the dimensions of the parallelepiped with a caliper, ensure that the zero is correctly set (Fig. 1.1).
2. Adjust the measuring jaws so that the parallelepiped fits securely between them.

3. The parallelepiped should be in contact with the surfaces of the measuring jaws.

4. Make sure the parallelepiped is securely fastened, without applying too much force.

5. Secure the vernier with the mounting screw

6. Take the caliper reading.

7. Measure each dimension L , H , and W of the parallelepiped four times. Enter the data into Tables 1.1, 1.2 and 1.3.

8. Using the algorithm for calculating direct measurement errors discussed in Section 3.4 of the Introduction, calculate the absolute and relative errors in measuring the length, height and width of the parallelepiped. Enter the results into Tables 1.1, 1.2 and 1.3.

9. Calculate the absolute error of the volume measurement, noting that it is an indirect measurement. Use the Error Propagation Formula (I.18) from Section 3.5 of the Introduction. For this experiment, the absolute error of V is obtained as follows:

$$\Delta V = LWH \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta W}{W}\right)^2 + \left(\frac{\Delta H}{H}\right)^2},$$

where ΔL , ΔW and ΔH are the total errors in L , W and H measurements.

Let us calculate the relative error of V for this experiment, using the formula:

$$\varepsilon_V = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta W}{W}\right)^2 + \left(\frac{\Delta H}{H}\right)^2}.$$

10. Write the final result as:

$$V = \bar{V} \pm \Delta \bar{V},$$

where $\bar{V} = \bar{L} \cdot \bar{H} \cdot \bar{W}$ is the mean value of the volume. \bar{V} is the product of the mean values of the length, height and width of the parallelepiped.

11. Write a report.

**Height measurement data and calculation of measurement errors
for H , W and L of a parallelepiped**

N	Heigh (H_i), mm	$H_i - \bar{H}$, mm	$(H_i - \bar{H})^2$, mm ²	$\Delta H_{i\text{ran}} = t_{p,N} \cdot S_H$ (random confidence error, P=95%)	Systematic error, $\Delta\theta_x$, sys
1					
2					
3					
4					
	$\bar{H} = (H_1 + H_2 + H_3 + H_4)/4$		$S_H = \sqrt{\frac{\sum_{i=1}^N (H_i - \bar{H})^2}{N(N-1)}}$	total absolute error $\Delta\bar{H} = \sqrt{\Delta H^2 + \Delta\theta_x^2}$ Relative Error $\varepsilon_H = \frac{\Delta\bar{H}}{\bar{H}} \times 100\%$	

Table 1.2

Width measurement data and calculation of measurement errors for W of a parallelepiped

N	Width (W_i), mm	$W_i - \bar{W}$, mm	$(W_i - \bar{W})^2$, mm ²	$\Delta W_{\text{ran}} = t_{P, N} \cdot S_W$ (random confidence error, P=95%)	Systematic error, $\Delta\theta_x$ sys
1					
2					
3					
4					
	$\bar{W} = (W_1 + W_2 + W_3 + W_4)/4$		$S_W = \sqrt{\frac{\sum_{i=1}^N (W_i - \bar{W})^2}{N(N-1)}}$	total absolute error $\Delta W = \sqrt{\Delta W^2 + \Delta\theta_x^2}$ Relative Error $\varepsilon_W = \frac{\Delta W}{W} \times 100\%$	

Table 1.3

**Length measurement data and calculation of measurement errors
for L of a parallelepiped**

N	Length (L_i), mm	$L_i - \bar{L}$, mm	$(L_i - \bar{L})^2$, mm ²	$\Delta L_{\text{ran}} = t_{p, N} \cdot S_L$ (random confidence error, P=95%)	Systematic error, $\Delta\theta_x$ sys
1					
2					
3					
4					
	$\bar{L} =$		$S_L = \sqrt{\frac{\sum_{i=1}^N (L_i - \bar{L})^2}{N(N-1)}}$	total absolute error $\Delta \bar{L} = \sqrt{\Delta L^2 + \Delta \theta_x^2}$ Relative Error $\epsilon_L = \frac{\Delta \bar{L}}{\bar{L}} \times 100 \%$	

Required sections of the report.

1. Main objective of the Lab.
2. Research methods and objects.
3. Theoretical framework indicating the main formulas and formulas for calculating the error.
4. Experimental data entered into a table.
5. Calculation of the volume of the parallelepiped, the errors of measuring its length, height and width, and the resulting error in the volume.
6. Conclusions indicating whether the task has been completed and presenting the final result.
7. Responses to the questions and problems.

Questions and problems.

1. What types of measurements exist?
2. What kinds of devices are used for measurements?
3. What is the accuracy class, and how can the absolute error be determined if the accuracy class is known?
4. How can the absolute error be determined if the accuracy class is unknown?
5. What is sensor sensitivity?
6. What is measuring range?
7. Determine the measurement range for the millimeter (a) and the device (b) shown in Figure I.3 (Introduction).
8. How is measurement accuracy determined?

Laboratory 2

Determining material density

The objective of the exercise is to determine the volume V of a cylinder and its mass, obtain the density of the cylinder material and calculate the errors of indirect measurements of the volume and density of the cylinder.

Instruments and accessories:

1. Cylinder.
2. Caliper and micrometer.
3. Scale.

Theoretical framework. Basic formulas.

The volume of a cylinder (Fig. 2.1) can be calculated using the formula:

$$V = \frac{\pi D^2}{4} H. \quad (2.1)$$

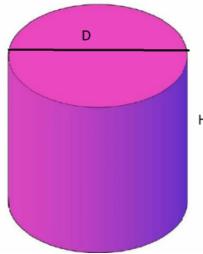


Fig. 2.1. A cylinder: $V = \frac{\pi D^2}{4} H$

An important property of the materials from which bodies are made is density. Density is a physical quantity that shows the amount of mass of a substance contained in a unit volume. It is numerically equal to the ratio of a body's mass to its volume:

$$\rho = m/V. \quad (2.2)$$

The SI unit of density is kg/m^3 . Knowing the geometric dimensions and mass of a body makes it possible to determine the material from which it is made using formula (2.2).

Procedure.

1. Measure the height H and diameter D of the cylinder. A vernier caliper or micrometer can be used in the process. Measure the cylinder's linear dimensions (H and D) four times.

Figure 2.2 shows the general view and structure of the micrometer and illustrates examples of measuring a body's size, demonstrating that it provides greater accuracy than a caliper. A distinctive feature of the micrometer is the presence of two scales with different division values: the main fixed scale consisting of two scales with divisions of 0.5 mm and 1 mm, and the scale applied on the rotating drum, with a division value of 0.5 mm. Note that two full revolutions of the drum correspond to a 1 mm shift. Therefore, readings on the micrometer must be taken carefully (Fig. 2.3, *a*, *b*).

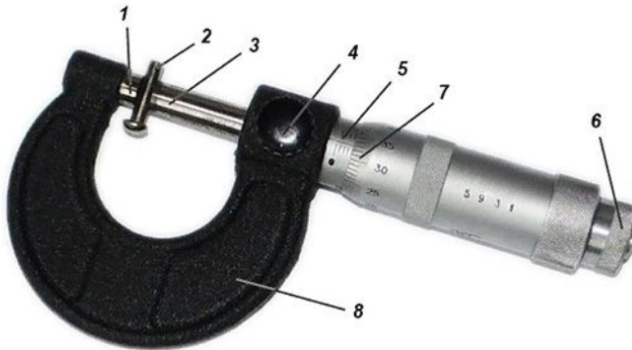


Fig. 2.2. General view of the micrometer:

1 — heel; 2 — object to be measured; 3 — micrometer screw spindle;
4 — stopper; 5 — horizontal fixed scale; 6 — ratchet; 7 — drum with a dial scale

2. Record the experimental measurement data in Tables 2.1 and 2.2.



a

b

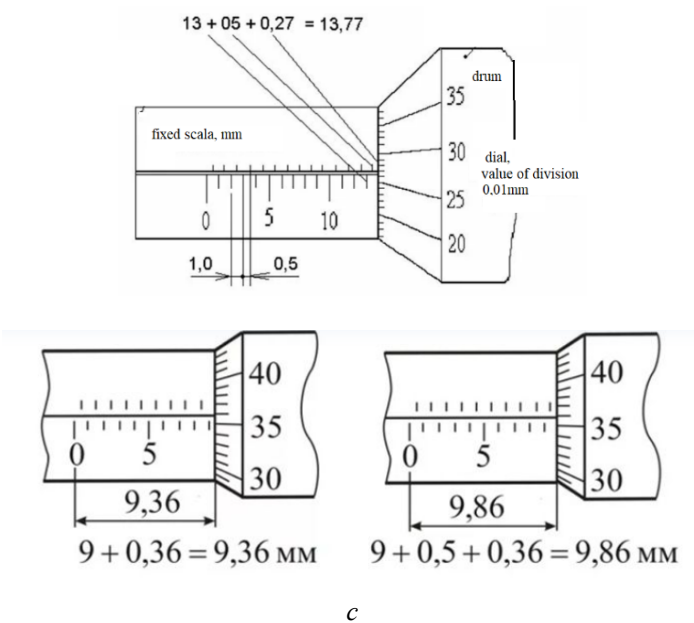


Fig. 2.3. Micrometer scales (*a*, *b*); examples of micrometer readings at different drum positions (*c*)

3. Using a precision scale, determine the mass of the cylinder and enter the data into Table 2.3. Calculate the accuracy of the scale, taking the smallest scale division as the absolute measurement error.

4. Calculate the errors of direct measurements, including the errors in measuring the height and diameter of the cylinder, and enter the data into Tables 2.1 and 2.2.

5. Calculate the volume of the cylinder by substituting the average height and diameter into formula (2.1).

6. Calculate the density of the cylinder by substituting the mean values of its diameter and mass into formula (2.2).

7. Analyse formula (I.18) for the error of indirect measurement of the cylinder's volume. The general formula for calculating the error of an indirect measurement is given in Section 4 of the Introduction:

$$\Delta F = \sqrt{\left(\frac{\partial F}{\partial x}\right)^2 \Delta x^2 + \left(\frac{\partial F}{\partial y}\right)^2 \Delta y^2 + \left(\frac{\partial F}{\partial z}\right)^2 \Delta z^2}.$$

In our case, the function $F=V$ is determined by only two measured parameters: diameter and height. Let us take the first derivatives of the function $V=\frac{\pi D^2}{4}H$ with respect to D and then with respect to H . The calculations yield the following result:

$$\frac{dV}{dD} = \frac{DH}{2}, \quad \frac{dV}{dH} = \frac{\pi D^2}{4}.$$

Let us substitute the obtained expressions into (I.18) and, after performing the transformations, obtain

$$\Delta V = \frac{\pi D^2}{4} H \sqrt{\left(2 \frac{\Delta D}{D}\right)^2 + \left(\frac{\Delta H}{H}\right)^2}. \quad (2.3)$$

8. Let us derive the formula for the error of measuring the cylinder's density. In this case, $F=\rho$, where density is determined by two parameters: mass and volume. As in the previous case, we apply formula (I.18) to obtain the error of an indirect measurement (Introduction, Section 4). We take the first derivative with respect to mass, then with respect to volume. The result is:

$$\frac{d\rho}{dm} = \frac{1}{V}, \quad \frac{d\rho}{dV} = \frac{-m}{V^2}.$$

Let us substitute the expressions into formula (2.3), obtaining:

$$\Delta\rho = \frac{m}{V} \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta V}{V}\right)^2}. \quad (2.4)$$

9. Calculate the error of indirect measurements and the total error of measuring the volume and density of the cylinder using formulas (2.3) and (2.4).

Calculate relative error as follows:

$$\varepsilon_V = \sqrt{\left(2\frac{\Delta D}{D}\right)^2 + \left(\frac{\Delta H}{H}\right)^2},$$

$$\varepsilon_\rho = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta V}{V}\right)^2}.$$

10. Identify the material of your cylinder, using Table 2.3.

11. Write the final result in the form:

$$V = \bar{V} \pm \Delta V,$$

Here \bar{V} is the mean value of volume.

$$\rho = \bar{\rho} \pm \Delta\rho.$$

Here $\bar{\rho}$ is the mean value of density.

12. Write a report.

Required sections of the report.

1. Main objective of the exercise.
2. Research methods and objects.
3. Theoretical framework indicating the main formulas and formulas for calculating the error.
4. Experimental data entered into Tables 2.1, 2.2 and 2.3.
5. Calculation of the volume of the cylinder, the volume error and the error of density measurement.
6. Conclusions indicating whether the task has been completed, presenting the final result and error calculations, and specifying the material the cylinder is made of.
7. Responses to the questions and problems.

Table 2.1

Height H measurement data and calculation of measurement errors for a cylinder

N	Height (H_i), mm	$H_i - \bar{H}$, mm	$(H_i - \bar{H})^2$, mm ²	$\Delta H_{\text{ran}} = t_{p,N} \cdot S_H$ (random confidence error, P=95%), mm	Systematic error, $\Delta\theta_x$ sys
1					
2					
3					
4					
	$\bar{H} = (H_1 + H_2 + H_3 + H_4)/4$		$S_H = \sqrt{\frac{\sum_{i=1}^N (H_i - \bar{H})^2}{N(N-1)}}$	total absolute error $\Delta H = \sqrt{\Delta H^2 + \Delta\theta_x^2}$ Relative Error $\varepsilon_H = \frac{\Delta H}{H} \times 100\%$	

Table 2.2

Measurement data for diameter D and calculation of its measurement errors for a cylinder

N	Diameter (D_i), mm	$D_i - \bar{D}$, mm	$(D_i - \bar{D})^2$, mm ²	$\Delta D_{\text{ran}} = t_{p,N} S_L$ (random confidence error, P = 95%), mm	Systematic error, $\Delta \theta_x$, sys
1					
2					
3					
4					
	$\bar{D} =$		$S_L = \sqrt{\frac{\sum_{i=1}^N (D_i - \bar{D})^2}{N(N-1)}}$	total absolute error $\Delta \bar{D} = \sqrt{\Delta D^2 + \Delta \theta_x^2}$ Relative Error $\varepsilon_L = \frac{\Delta \bar{D}}{\bar{D}} \times 100 \%$	

Table 2.3

**Mass measurement data and calculation
of density measurement errors for a cylinder**

N	mass, g	Systematic error, sys	\bar{V} mean, volume	ΔV total volume error, mm^3	$\bar{\rho} = (m/\bar{V})$, g/cm^3 , density of the cylinder
1					
					Relative Error $\varepsilon_{\rho} = \frac{\Delta\rho}{\bar{\rho}} \times 100\%$

Questions and problems.

1. What is the density of a substance? Which substance on Earth has the highest density (see Table 2.4)?
2. Derive a formula for calculating the relative error of a substance's density if the relative errors of mass and volume are already known.
3. Will the relative error of a physical quantity change if the number of experiments in direct measurements is increased?
4. Will the error of an indirect measurement of the cylinder's volume change if the number of measurements of its height and diameter is increased?
5. Explain why the accuracy of measurements with a micrometer is higher than with a caliper.

Table 2.4

Density of materials

Material	Density (g/cm^3)	Density (kg/m^3) $\times 10^3$
Hydrogen (at STP, gas)	0.00009	0.00009
Nitrogen (at STP, gas)	0.001251	0.001251
Air (at STP, gas)	0.001293	0.001293

Continuation of the Table 2.4

Material	Density (g/cm ³)	Density (kg/m ³) × 10 ³
Lithium (metal), Li	0.53	0.53
Sodium (metal), Na	0.97	0.97
Ice, H ₂ O	0.920	0.920
Water at 20 °C (liquid), H ₂ O	0.998	0.998
Water at 4 °C, H ₂ O	1.000	1.000
Seawater	1.03	1.03
Milk	1.03	1.03
Coal	1.1—1.4	1.1—1.4
Rubidium (metal), Rb	1.52	1.52
Blood	1.600	1.600
Magnesium, (metal) Mg	1.7	1.7
Beryllium, (metal) Be	1.84	1.84
Silicon	2.55	2.55
Granite	2.6—2.7	2.6—2.7
Aluminium (metal), Al	2.7	2.7
Diamond	3.47—3.55	3.47—3.55
Titanium (metal), Ti	4.5	4.5
Germanium (metal), Ge	5.33	5.33
Gallium (metal), Ga	5.91	5.91
Tellurium (metal), Te	6.25	6.25
Zirconium (metal), Zr	6.53	6.53
Vanadium (metal), V	6.5—7.1	6.5—7.1
Zinc (metal), Zn	7.13	7.13
Chrome (metal), Cr	7.14	7.14
Manganese (metal), Mg	7.2—7.4	7.2—7.4
Tin (metal), Sn	7.3	7.3
Indium (metal), In	7.36	7.36
Steel	7.8	7.8
Iron, Fe	7.8	7.8
Cadmium (metal), Cd	8.64	8.64
Cobalt (metal), Co	8.9	8.9
Nickel (metal), Ni	8.9	8.9

End of the Table 2.4

Material	Density (g/cm ³)	Density (kg/m ³) × 10 ³
Copper, Cu	8.3—9.0	8.3—9.0
Bismuth (metal), Bi	9.8	9.8
Molybdenum (metal), Mo	10.3	10.3
Silver (metal), Ag	10.5	10.5
Lead (metal), Pb	11.3	11.3
Thallium (metal), Tl	11.85	11.85
Palladium (metal), Pd	12.0	12.0
Ruthenium (metal), Ru	12.45	12.45
Rhodium (metal), Rh	12.48	12.48
Hafnium (metal), Hf	13.09	13.09
Mercury (metal), Hg	13.6	13.6
Tantalum (metal), Ta	16.6	16.6
Uranium, U	18.7	18.7
Tungsten (metal), W	19.3	19.3
Gold (metal), Ag	19.3	19.3
Rhenium (metal), Re	21.0	21.0
Platinum (metal), Pt	21.2—21.5	21.2—21.5
Osmium, Os	22.6	22.6
Iridium, Ir	22.4—22.6	22.4—22.6
White dwarf star	10 ⁷	10 ⁷

Laboratory 3

Atwood machine: investigation of the rectilinear motion of bodies in a gravitational field using the Atwood machine

The objective of this exercise is to study uniformly accelerated motion experimentally and to determine gravitational acceleration using the **Atwood machine**.

Instruments and accessories:

1. Atwood machine with a set of additional weights of types A and B.
2. Stopwatch.
3. Electromagnet with a button starting the motion of the system (E).
4. Measuring ruler (K).

Theoretical framework. Studying the motion of rolling bodies, Galileo concluded that freely falling bodies move with constant acceleration. The main difficulty in accurately determining gravitational acceleration lies in its large magnitude. Thus, the experiment requires either a very tall apparatus (such as the Leaning Tower of Pisa) or special methods capable of precisely measuring very short time intervals (fractions of a second). The Atwood machine overcomes these difficulties and makes it possible to study the law of motion of bodies in the Earth's gravitational field.

Atwood machine.

The principal part of the Atwood machine is a system of moving bodies, consisting of a pulley with a thread H passing over it, to the ends of which equal masses m are attached. The system is driven by an additional weight m_0 placed on one of the masses m (see Fig. 3.1). The position of a mass m is measured using scale K . When one mass is at the top position, the other is held by the electromagnet E . Ring platform C_1 and solid platform C_2 can move

along a vertical column (rack) marked with scale K. A set of additional masses of types A and B is also provided (Fig. 3.1). Type A additional weights pass freely through the aperture in platform C_1 , while type B additional weights are stopped by ring-shaped platform C_1 as the mass m moves, staying on the platform.

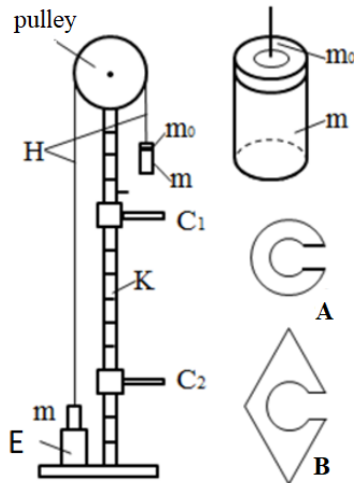


Fig. 3.1. Diagram of the Atwood machine

Thin type A additional weights (less than 1 mm thick) are used to compensate for **friction forces** (see Task 1) and their mass is usually negligible. Thicker type A additional weights are used to create accelerated motion of the system. One or more additional weights can be added to mass A, causing the system to become unbalanced and start moving rapidly.

Obtaining formulas for the calculations.

Let us determine the law of motion of the masses. In the calculations, a fixed coordinate system is used, with its origin coinciding with the axis of the pulley. The OX axis is directed downward.

Consider the forces acting on the masses. An additional weight m_0 is added to the mass on the right. As the masses move, they are acted upon by gravitational forces mg and $(m+m_0)g$, and by the tensions in the thread, T_1 and T_2 (see Fig. 3.2). According to Newton's second law, the equations governing the translational motion of the have the form:

$$mg - T_1 = -ma_1,$$

$$(m + m_0)g - T_2 = (m + m_0)a_2, \quad (3.1)$$

where T_1 is the tension force, a_1 and a_2 are the accelerations of the masses, and g is the acceleration due to gravity. Expression (3.1) includes the projections of the above forces and accelerations on the vertical axis.

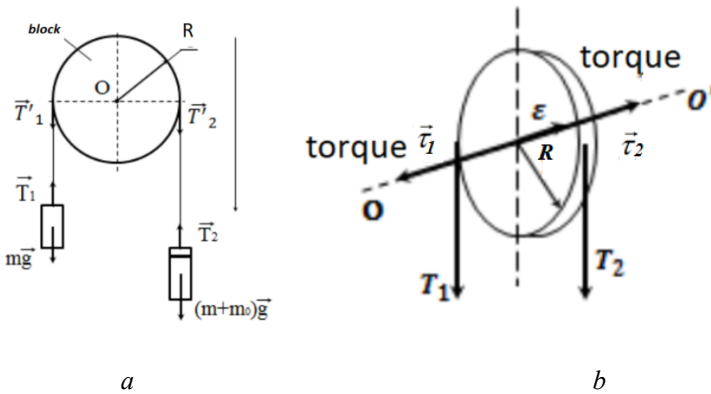


Fig. 3.2. Forces acting on the disk (a) and forces causing torque (b)

Due to the inextensibility of the thread, the acceleration of the mass on the right is equal in magnitude and opposite in direction to the acceleration of the mass on the left, therefore,

$$a_1 = a_2 = a.$$

The tension at the right end of the thread is denoted by T_2 .

Let us write the basic equation for the dynamics of the rotational motion of rigid bodies relative to the pulley. The sum of the torques is equal to the product of the moment of inertia and the angular acceleration:

$$\sum_{i=1}^n \vec{\tau}_i = I\vec{\beta}, \quad (3.2)$$

where $\sum_{i=1}^n \vec{\tau}_i$ is the total torque about the axis of rotation attached to the pulley; I is the moment of inertia of the rotating body; $\vec{\beta}$ is the angular acceleration, where R is the radius of the pulley (Fig. 3.2).

Angular acceleration is related to linear acceleration a as follows:

$$\beta = \frac{a}{R}. \quad (3.3)$$

Note that the pulley surface is directly affected by the static **friction forces** exerted by the deformed thin thread that is in contact with the surface of the pulley (Fig. 3.3) and remains stationary relative to it. The mass of the thread in contact with the pulley surface is very small, and therefore the moment of inertia of this part of the thread, I_1 , is effectively zero (more precisely, $I_1 \ll I$). The resulting moment of the static friction forces produces its angular acceleration:

$$\tau_{fr} = \tau_2 - \tau_1.$$

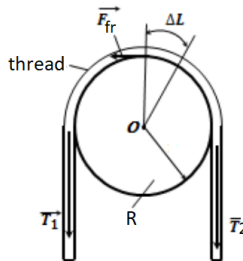


Fig. 3.3. The surface of the pulley directly affected by static **friction forces** exerted by a deformed thin thread

For the pulley, taking into account the last two expressions, the fundamental law of the dynamics of rotational motion can be written as:

$$(T_1 - T_2)R = I \frac{a}{R}. \quad (3.4)$$

R is the pulley radius $R=(0.066 \pm 0.001)$ m. The moment of inertia of the circular disk is given by $I = \frac{M_0 R^2}{2}$, where M_0 is the mass of the pulley (its circular disk).

From the system of equations (3.1), (3.2) and (3.4), we find the linear acceleration:

$$a = \frac{m_0}{2m+m_0+\frac{M_0}{2}} g, \quad (3.5)$$

where M_0 is the mass of the pulley, $m_0=(115.0 \pm 0.5)$ g; $m=(161.0 \pm 0.5)$ g is the mass of weights A and B; $m_0=0.2$ g (determined experimentally).

Thus, the motion of object A is uniformly accelerated and follows equation (3.5), which could be used to determine the acceleration g . However, the experiment is complicated by the lack of simple methods for directly measuring the acceleration a . To determine a , the laws of uniformly accelerated motion are applied. The acceleration can be found by measuring the path S and the time t of the object's motion ($v_0=0$). These quantities are related by the standard equation:

$$S = \frac{at^2}{2}. \quad (3.6)$$

From equation (3.6), the acceleration a can be expressed as:

$$a = \frac{2S}{t^2}. \quad (3.7)$$

Combining formulas (3.7) and (3.5) yields:

$$t^2 = \frac{2S(2m+m_0+\frac{M_0}{2})}{gm_0}. \quad (3.8)$$

Experimental part. Laboratory exercise steps.

1. Friction compensation in the system.

Determine m_0 experimentally. To do this, milligram weights are placed on mass A, gradually increasing the load until the mass begins to descend.

2. Friction compensation is obtained as follows:

1. Place a type B object on the right-hand mass, causing the masses to move rapidly.

2. Set the system to the initial position so that the left-hand mass is at the bottom, held by an electromagnet (Fig. 3.1).

3. Install ring-shaped platform C_1 below the right-hand mass at a distance ℓ from the lower base of the mass (Fig. 3.1). The value of ℓ is specified in the apparatus manual.

4. Open the electromagnet circuit and, during the motion of the masses, measure the time t from the moment the type B additional weight is stopped by ring platform C_1 until the right-hand mass reaches solid platform C_2 .

5. Measure the distance travelled by the right-hand mass after the type B additional weight has been halted. The path S equals the distance X between the ring-shaped and solid platforms minus the height of the right-hand mass, h .

6. Adjust the solid platform to a new position and repeat the measurements described in steps 3—5.

The experiment is repeated three times in total, after which the ratios S_1/t_1 are calculated. If friction is compensated, the masses will move uniformly after the type B additional weight is returned to the mass as it passes upward through the ring-shaped platform.

$$v = \text{const.}$$

Differences in the magnitude of the ratios of up to 10% are permitted. If the ratios are not equal, the experiment is repeated from the beginning by adding or removing one or more light additional weights of type A on the right-hand mass.

3. Study of the rectilinear motion of bodies in a gravitational field. The experiment is conducted in the following sequence.

1. Without removing the friction-compensating additional weights from the right-hand mass, add type A additional weights (more than 1 mm thick). Record the mass of a type A additional weight in Table 3.1. The mass of the weight is indicated on its surface. Mass A should be in contact with the electromagnet (the current in the electromagnet coil must be switched on in advance).

Table 3.1

Experimental data for calculating the acceleration of gravity

i	Additional weight, m_0 , (g)					Additional weight, m_0 , (g)				
	S_1 m	t_i s	t_{mean} , s	t_{mean}^2 s^2	a_1 m/s^2	S_1 m	t_i s	t_{mean} , s	t_{mean}^2 s^2	a_2 , m/s^2
1 : 5										
	S_2 m	$t_{i,S}$	t_{mean} , s	t_{mean}^2 s^2	a_1 m/s^2	S_2 m	t_i s	t, s	t_i^2 , s^2	a_2 , m/s^2
1 : 5										
	S_3 m	t_i s	t_{mean} , s	t_{mean}^2 s^2	a_1 m/s^2	S_3 m	t_i s	t_{mean} , s	t_{mean}^2 s^2	a_2 m/s^2
1 : 5										

2. Set the stopwatch to zero. Raise the platform until it comes into contact with mass A, and mark the initial height of the load, h_0 , on the scale. Then lower the platform by a distance S.

3. Break the electromagnet circuit and start the stopwatch simultaneously.

4. When mass A comes into contact with the platform, stop the stopwatch and record the time taken for the mass to descend. Enter the experimental data S and t into Table 3.1.

5. Knowing S and t , calculate a using formula (3.7).

Repeat the experiment from three to five times, recording the data into Table 3.1.

Before proceeding with systematic measurements, it is advisable to perform several preliminary experiments at different values of S and t to verify the correct operation of the apparatus. The value of g calculated from the experimental data using formula (3.5) should be compared with the standard value.

Procedure.

1. Place additional weights on mass A and measure the time required to travel the distance S , performing the procedure at least five times.

2. Repeat the experiment with different additional weights: $m_0 = 2\text{g}, 4\text{g}$.

3. Repeat the experiment for three different platform positions — $S_1, S_2,$ and S_3 . Record the data in the table.

4. Plot a graph of $t^2 = f(S)$ using the experimental data. Verify the uniformly accelerated nature of the motion. The slope (k) of the graph (3.8) can be used to determine the gravitational acceleration according to formula (3.9).

$$t^2 = \kappa S, g = \frac{2(2m+m_0+\frac{M_0}{2})}{km_0}. \quad (3.9)$$

5. Calculate the values of the acceleration a for each additional weight. Determine the average value and the errors in measuring a for each weight (3.10).

6. Using the calculated values of a , determine the gravitational acceleration g and compare it with the standard value. Assess the accuracy of the results obtained.

$$\Delta a = \bar{a} \sqrt{\left(\frac{\Delta S}{S}\right)^2 + \left(\frac{\Delta t}{t}\right)^2}, \quad (3.10)$$

$$\Delta g = \bar{g} \sqrt{\left(\frac{\Delta a}{\bar{a}}\right)^2 + \left(\frac{\frac{\Delta m_0}{2} + 2\Delta m}{m_0 + \frac{M_0}{2} + 2m}\right)^2}. \quad (3.11)$$

7. Prepare a report. Include in the report the completed Table 3.1, the calculations of the accelerations of the weights, the determined gravitational acceleration and the errors in measuring the accelerations. Answer the questions below.

Questions and problems.

1. Formulate and write down Newton's second law.
2. What is a reference system? In which situations can the Earth be used as a frame of reference and in which situations can it not?
3. What is a material point? Provide examples. Under what conditions can the Earth be treated as a material point?
4. What is the difference between the concepts of path and displacement?
5. What is average and instantaneous speed? Can average and instantaneous speeds coincide and in what cases?
6. Determine the average and instantaneous acceleration. Can average and instantaneous accelerations coincide and in what cases?
7. Provide the path formulas for uniform linear motion.
8. What is force? In what units is force expressed?
9. What does body weight measure?
10. Formulate the basic equations of dynamics for translational and rotational motion (Newton's laws).
11. What is an isolated system? Provide examples.
12. What motion of a body is called free fall?
13. How does the gravitational acceleration change when a body rises from sea level to a height of $h=10$ km? The radius of the Earth is $R_e=6.370$ km. Perform calculations in the SI system.
14. Compare the gravitational acceleration on Earth and Mars.
15. Under what condition do the accelerations of the bodies used in the Atwood machine have the same magnitude?

16. What is the moment of inertia of a body? How does the acceleration of the body change if the rotation of the disk is neglected?

17. What is linear and angular acceleration if a body rotates in a circle? Write down a formula expressing the relationship between linear and angular velocities.

18. Will the tension in the thread change during the motion of the masses if one additional weight is replaced by another?

Laboratory 4

Determination of bullet velocity by the physical pendulum method

This **lab** demonstrates the laws of conservation in mechanics.

The **objective** of this exercise is to study the conservation laws of total mechanical energy and angular momentum. When applying these laws, it is essential to account for changes in total mechanical energy during an inelastic collision.

In this exercise, the speed of a spring-gun bullet is determined from the deviation of a physical pendulum from its equilibrium position.

Instruments and accessories.

A laboratory setup consisting of a physical pendulum.

Setup dimensions:

The length does not exceed 470 mm.

The width does not exceed 210 mm.

The height does not exceed 670 mm.

The weight does not exceed 7 kg.

The bullet mass is $m_1 = (2.4 \pm 0.03)$ g.

The rod mass is $m_2 = (50.0 \pm 0.5)$ g.

The trap mass is $m_3 = (12.5 \pm 0.5)$ g.

The distance from the axis to the centre of the trap is $L_1 = (575 \pm 0.5)$ mm.

The length of the rod is $L_2 = (570 \pm 0.5)$ mm.

The distance from the axis to the ruler is $L' = (625 \pm 0.7)$ mm.

Experimental setup (Fig. 4.1).

1. A spring-loaded pistol is fixed to the base (1). Its fixed latch part is equipped with a ruler and a limiter.

2. The support (2) that carries the axis of the physical pendulum consists of a rod (3) and a bullet trap (catcher) (4).

3. The trap is fitted with a pointer fixed to it (5) and the movable part of the pendulum with an end-position lock (6).

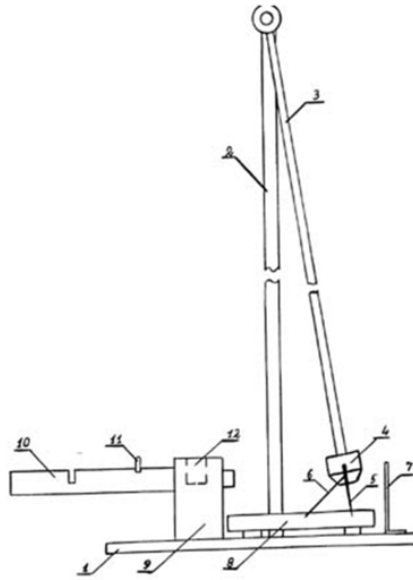


Fig. 4.1. A physical pendulum

The setup also includes a pendulum limiter (7) and a fixed end-position lock with a measuring ruler (8) and the spring pistol. The spring pistol comprises a pistol base (9), a cylinder with a spring (10) and a handle (11) for compressing the spring, holding it in the compressed position and firing. A rectangular opening (12) in the upper part of the pistol base is used to load a cylindrical bullet supplied for the experiments.

Theoretical framework. Derivation of the formula for bullet speed.

When deriving the calculation formula, the absolutely inelastic collision of a bullet with a physical pendulum should be considered. Upon interacting with the pendulum, the bullet is inelastically

slowed, and the pendulum acquires an angular velocity. As a result of the collision, the pendulum deviates by an angle α from the vertical axis. If the duration of the bullet's impact is small compared to the oscillation period T of the pendulum, the pendulum does not have time to deviate significantly from its initial position during the impact. Furthermore, since the moment of external forces is small (external forces are much weaker than internal ones), the bullet-pendulum system can be treated as quasi-closed, and the law of conservation of angular momentum (E.11), (E.12) can be applied.

$$m_1 v L_1 = I \omega, \quad (4.1)$$

where m_1 is the mass of the bullet, v is the bullet's speed, L_1 is the distance from the pendulum axis to the point of impact, and I is the moment of inertia of the pendulum with the bullet relative to the axis of rotation of the physical pendulum. In this case, the moment of inertia is given by:

$$I = (m_2 L_2^2)/3 + (m_1 + m_3) L_1^2, \quad (4.2)$$

where m_1 is the mass of the bullet, m_2 is the mass of the rod, m_3 is the mass of the trap, and L_2 is the length of the rod.

A physical pendulum, with an initial angular velocity ω , is deflected by an angle α (ballistic throwback). As the pendulum rises, its centre of mass ascends to a height h . The law of conservation of mechanical energy after the impact can be expressed as:

$$I \omega^2 / 2 = (m_1 + m_2 + m_3) g h, \quad (4.3)$$

where

$$h = R(1 - \cos \alpha) = 2R \sin^2(\alpha/2) \quad (4.4)$$

h is the height of the centre of mass when the pendulum is deflected, R is the distance from the point of suspension of the pendulum to the centre of gravity of the system:

$$R = (m_2 L_2 / 2 + (m_1 + m_3) L_1) / (m_1 + m_2 + m_3). \quad (4.5)$$

Expressing v from (4.1) yields:

$$v = \omega I / m_1 L_1. \quad (4.6)$$

The angular velocity ω can be obtained from equation (4.3) as:

$$\omega = [2gh(m_1 + m_2 + m_3)/I]^{1/2} \quad (4.7)$$

therefore,

$$v = (l/m_1 L_1) [2ghI(m_1 + m_2 + m_3)]^{1/2}. \quad (4.8)$$

Substituting the values h and I into (4.8) gives:

$$V = (2\sin\alpha/2)([g(m_2 L_2/2 + m_1 L_1 + m_3 L_1) \times \\ \times (m_2 L_2^2/3 + m_1 L_1^2 + m_3 L_1^2)]^{1/2})/m_1 L_1.$$

Assuming $m_1 \ll m_2$ and m_3 , and $L_1 \approx L_2 = L$, we obtain:

$$V = (\sin\alpha/2) [2gL(m_2^2 + 5m_2(m_1 + m_3) + 6(m_1 + m_3)^2)/3]^{1/2}/m_1. \quad (4.9)$$

Since the angle α is small, $\sin(\alpha/2)$ can be approximated by $\alpha/2$ (in this case, the angle must be expressed in radians), where:

$$\alpha = (S - S_0)/L',$$

where L' is the distance from the axis of rotation of the pendulum to the ruler, S_i is the value of the pointer position after the shot and S_0 is the initial position of the pointer.

Preparing the setup for operation.

1. Attach the support carrying the physical pendulum to the base. Ensure the slot in the movable part of the latch overlaps the fixed part so the pendulum moves along the ruler without friction.
2. If necessary, adjust the spring pistol so the bullet strikes the centre of the trap aperture.

Experimental part. Laboratory exercise steps.

1. Weigh the bullet on the scales and determine its mass m .
2. Record the setup component data (see **Instruments and accessories**): $m_1 = \dots m_2 = \dots m_3 = \dots L_1 = \dots L_2 = \dots L' = \dots$
3. Using handle 11 (Fig. 4.1) compress the gun spring and lock it by turning the handle counter-clockwise.
4. Raise the movable part of latch 6 on the trap and move the pendulum to the vertical position.

5. Record the initial position of pointer S_0 .
6. Insert a cylindrical bullet into the pistol through slot 12 in the base.
7. Fire the shot by turning the handle clockwise.
8. Record the position of pointer S in Table 4.1. Repeat the experiment at least five times.

Table 4.1

The position of the pointer S , initial position S_0 and angle α

No. n/n	1	2	3	4	5	S_{mean}	$S_m - S_0$	α_{mean}
S, mm								

9. Determine the average value of the angle α_{mean}

$$\alpha_{mean} = (S_{mean} - S_0) / L'$$

10. For each measurement, calculate the bullet velocity V using formula (4.9). Values L_1 , m_1 and m_2 are given at the start of the laboratory exercise description.

11. Calculate the error using the formula (I.18):

$$((\Delta V/V) = [(\Delta\alpha/\alpha)^2 + (\Delta m_1/m_1)^2 + 0.25[(\Delta l/l)^2 + ((2m_2 + 5m_3)^2 \Delta m_2^2 + (5m_2 + 12m_3)^2 \Delta m_3^2)] / (m_2^2 + 5m_2m_3 + m_3^2)]^{1/2}.$$

12. Write the final result as:

$$V = (\bar{V} \pm \Delta V).$$

Additional task: according to the experiment, determine the loss of mechanical energy in a completely inelastic impact.

Required sections of the report.

1. Main objective of the exercise.
2. Research methods and objects.
3. Theoretical framework indicating the main formulas and formulas for calculating the error.

4. Experimental data and the calculated bullet velocity entered into Table 4.1.

5. Conclusions indicating whether the task has been completed and presenting the final result.

6. Responses to the questions and tasks.

Questions and tasks.

1. Describe the stages of the bullet speed experiment at which the law of conservation of momentum and the law of conservation of energy are satisfied.

2. What is the moment of inertia of a rigid body? Formulate the law of conservation of angular momentum and specify the conditions under which it is valid.

3. Formulate the Huygens — Steiner theorem (E.3).

4. Write the formula for the period of oscillation of a pendulum (mathematical and physical pendulum).

5. Explain the principle of measuring the speed of a projectile using a physical pendulum and derive the formula for determining the bullet speed.

6. How would the angle of deflection of the pendulum change if the impact were considered absolutely elastic rather than absolutely inelastic?

Labratory 5 Determination of the resistivity of a wire

The **objective** of this exercise is to investigate Ohm's law and the practical application of the laws of direct current in calculating the resistivity of a wire and in identifying its material.

Instruments and accessories:

1. Laboratory setup with an ammeter-voltmeter.
2. Micrometer.

Theoretical framework. Ohm's law.

According to Ohm's law, the current flowing through a conductor is directly proportional to the voltage applied. The proportionality constant is $1/R$, where R is the electrical resistance of the conductor.

$$I = \frac{U}{R}. \quad (5.1)$$

The resistance R of a component in a circuit is given by the equation $U=IR$, where U is the potential difference across the component and I is the current in the component.

The resistance of a conductor can be determined if the voltage across it and the current flowing through it are known. The resistance of a wire is expressed by the equation

$$R = \rho L / A, \quad (5.2)$$

where ρ is the resistivity of the metal from which the wire is made, L is the length of the wire and A is its cross-sectional area. However, difficulties arise in measuring resistance, as both current and voltage must be known simultaneously. Let us consider two circuits for measuring resistance. In the first circuit (Fig. 5.1, *a*), the voltmeter gives the correct potential difference across the resistance. Yet the ammeter reading will be inaccurate, since the current in the circuit splits: part flows through the voltmeter and part through the resistor.

In the second circuit diagram (Fig. 5.1, *b*), the ammeter shows the correct current through the resistor, but the voltmeter gives an erroneous value, since it measures the total voltage drop across both the resistor and the ammeter.

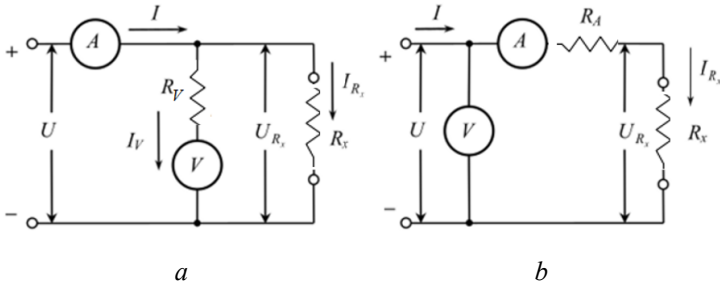


Fig. 5.1. Circuits for measuring active resistance:
a — voltmeter method; *b* — ammeter method

To determine the resistance of the wire by the voltmeter-ammeter method, the laboratory setup shown in Figure 5.2 is used.



Fig. 5.2. A laboratory setup

Determining the resistance of a wire by the voltmeter–ammeter method involves measuring the current I through the wire and the voltage drop U across it. Resistance R is calculated using Ohm’s law (5.1). The laboratory setup above allows two possible options for connecting the electrical measuring instruments used in the exercise. The choice of a particular circuit for connecting the instruments is made using the switch mode button, which has two positions, Sx-1 and Sx-2, corresponding to the button being released or pressed.

Measuring resistance by ammeter-voltmeter method.

Step 1

Check the setup. Determine the scale division and the accuracy class of the ammeter and voltmeter. Enter the data into Table 5.1.

Table 5.1

Tool parameters

Device	Accuracy class	Scale division	Internal resistance
Micrometer	(mm)	—
Ammeter	(mA)	$R_A=0.15 \Omega$
Voltmeter	 (V)	$R_U=2500 \Omega$

Step 2

1. Determine the diameter of the wire using a micrometer five times: $D_1 = \dots$ mm, ..., $D_5 = \dots$ mm.

2. Calculate the cross-sectional area of the wire

$$A = D^2/4.$$

3. Determine the length of the wire, L , and record it in Table 5.2.

4. Measure the current I and voltage U in both operating modes of the unit, repeating the measurements five times.

5. Determine the resistance by measuring the conductor’s cross-sectional area and length, using a micrometer to measure the cross-section.

6. Write the data in Table 5.2.

Table 5.2

Experimental data for calculating wire resistivity

$L,$ m	Scheme 1: button released			Scheme 2: button pressed			R, Ω			$\rho,$ Ωm	$\Delta\rho$ Ωm
	$I,$ A	$U,$ V	$R_{ex},$ Ω	$I,$ A	$U,$ V	$R_{ex},$ Ω	Scheme 1	Scheme 2	R, Ω		

Step 3

1. Calculate the resistance using the current and voltage values obtained by the first method, corresponding to the released-button position (scheme 1).

$$R_{ex} = U / I.$$

2. Find the resistance of the wire:

$$R_1 = R_{ex} (1 - R_a / R_{ex}).$$

3. Calculate the resistivity of the wire using the first method.

$$\rho_1 = R_1 A / L.$$

4. Calculate the resistance using the current and voltage values obtained by the second method, with the button pressed (scheme 2).

$$R_{ex} = U / I.$$

5. Find the resistance of the wire using the second method.

$$R_2 = R_{ex} (1 - R_{ex} / R_v).$$

6. Calculate the resistivity of the wire using the second method.

$$\rho_2 = R_2 A / L.$$

7. Calculate the mean value of the resistivity ρ and record all data in Table 5.2.

8. Estimate the relative error in measuring the resistivity, using the following formula:

$$\frac{\Delta\rho}{\rho} = \sqrt{\left(\frac{\Delta U}{U}\right)^2 + \left(\frac{\Delta l}{l}\right)^2 + \left(\frac{\Delta l}{l}\right)^2 + \left(2\frac{\Delta d}{d}\right)^2}. \quad (5.3)$$

To calculate the error (5.3), refer to the accuracy class of the instruments.

9. Estimate the absolute error $\Delta\rho$ and write the final result as:

$$\rho = \rho_{mean} \pm \Delta\rho.$$

10. Determine the material of the wire, using resistivity tables.

11. Prepare a report.

Questions and tasks.

1. Formulate Ohm's law.

2. Explain why it is difficult to determine the resistance of a wire accurately using a single measurement with an ammeter and a voltmeter.

3. Which law states that the resistance of a conductor changes with temperature?

4. What are the resistivities of different tissues in a living organism? Give examples.

Laboratory 6

Adjusting the current with a rheostat and verifying Ohm's law

The **objective** of this exercise is to experimentally check Ohm's law and establish the relationship between current and voltage.

Instruments and accessories:

1. Current source.
2. Ammeter.
3. Voltmeter.
4. Rheostat.
5. Key and connecting wires.

Follow the safety rules.

- Do not touch bare wires with wet hands.
- Do not alter the circuit.
- Before changing the position of the slider, turn off the circuit with the key.
- The second rheostat in the circuit is intended to prevent short circuits; do not move its slider.

Procedure.

1. Draw a measurement diagram, verify Ohm's law and determine the dependence of current on the voltage across the resistor (Fig. 6.1).

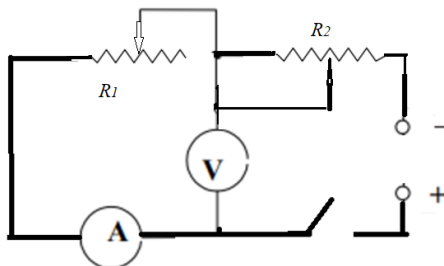


Fig. 6.1. Setup scheme

2. Note that the device scale is determined by the switch. Determine the scale division of the ammeter and voltmeter according to the switch readings.

Before taking measurements, determine the scale division of the ammeter and voltmeter and record it in Table 6.1. The scale division is defined by the smallest interval between marks. First, identify the measurement limit (the maximum value on the instrument scale), then count the number of divisions.

Table 6.1

Device characteristics

Metrological Characteristics	Ammeter	Voltmeter
Scale division	$h_{mA} = \dots (\dots)$	$h_V = \dots (\dots)$
Nominal scale value	$X_{max} = \dots (\dots)$	$X_{max} = \dots (\dots)$
Accuracy class	$AC = \dots$	$AC = \dots$

3. In the example shown in Figure 6.2, the maximum value on the ammeter scale is 100 A, with 50 divisions, giving a scale division of 2 A.

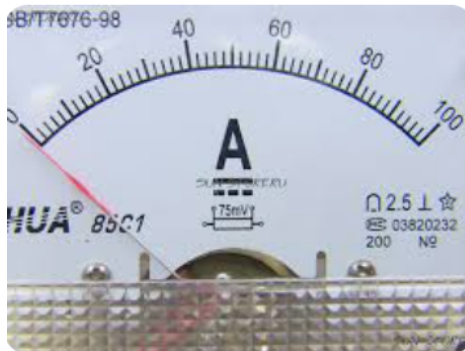


Fig. 6.2. An ammeter

4. Set the resistance on the rheostat, close the key, record the readings and enter the current and voltage values into Table 6.2.

Table 6.2

Experimental data: current and voltage

N	U (v)	I (A)	R(Ω)
1			
.			
.			
10			

5. Unlock the key, set the next resistance value, measure the current and voltage in the circuit and record the values in Table 6.2
6. Plot a graph showing the dependence of current on voltage.
7. Calculate the resistance from the current and voltage data in Table 6.2 using Ohm's law.

Required sections of the report.

1. Main objective of the exercise.
2. Research methods and objects.
3. Completed Tables 6.1 and 6.2 indicating the characteristics of the devices and the calculated resistance values.
4. A graph demonstrating Ohm's law.
5. Conclusions.

Questions.

1. How is Ohm's law formulated for a section of a circuit and for a complete circuit?
2. What is electric current, and what parameters does the current in a conductor depend on?
3. Under what conditions can an electric current arise?
4. What parameters determine the resistance of a conductor?
5. How does the resistance of a conductor change with temperature?
6. What is the accuracy class of an ammeter or voltmeter?
7. Why does a conductor heat more during extended use?

8. Which law determines the amount of heat released when current passes through a conductor?
9. What is the direction of current? In which direction do electrons move in a conductor and ions in an electrolyte?
10. What effect does current have on tissues and the body as a whole?
11. What factors determine the danger of electric shock?
12. Which current is more dangerous for the body, alternating or direct?
13. What kind of conductor connection is called series?
14. What kind of conductor connection is called parallel?
15. How is the equivalent resistance calculated for parallel and series conductor connections?
16. What are the maximum permissible values of current and voltage?

Laboratory 7

Determination of the viscosity coefficient of a liquid using Stokes' method

The **objective** of this exercise is to become familiar with the laws of hydrostatics and hydrostatic quantities and to study the method for determining the viscosity coefficient of a liquid.

Instruments and accessories:

1. Glass cylindrical vessel filled with a viscous liquid.
2. Ruler.
3. Stopwatch.
4. Microscope.
5. Spherical lead pellets.

Theoretical framework.

Experimental setup description.

The experimental setup for studying the motion of bodies in a viscous liquid consists of two glass cylindrical vessels filled with viscous liquids. Two marks are placed on the vessel, upper and lower, with the distance between them equal to L . According to the professor's instructions, a series of experiments are carried out with one of the liquids. Three to five small balls are lowered into the vessel one by one; each ball has a density ρ_1 greater than the density of the liquid ρ_2 . The diameters of the balls are measured beforehand using a microscope, where each ball appears as a circle through which a marked scale passes. The centre of the circle should be aligned as precisely as possible with the scale line, and readings should be taken from the left and right edges of the circle. The difference in readings, multiplied by the microscope's accuracy coefficient, gives the value of the ball diameter. For greater accuracy and to compensate for the irregular shape of the lead pellets, repeat the measurements three times, rotating each ball.

The density of the pellet material is $\rho_1 = 11300 \text{ kg/m}^3$.

The density of the liquid is $\rho_2=960 \text{ kg/m}^3$ (castor oil) or $\rho_2=1260 \text{ kg/m}^3$ (glycerin).

If the diameter of the balls, the distance between the two marks (A and B) (Fig. 7.1) and the time taken for each ball to move between them are measured experimentally, the viscosity of the liquid can be determined using the Stokes method.

Stokes method.

Let us consider the forces acting on a ball in the liquid (Fig. 7.1). These include the gravity force $F_T=\rho_1gV_1$, directed downwards ($V_1=1/6 \pi d^3$ is the volume of the ball and d is the diameter of the ball), the Archimedes force $F_A=\rho_2gV_1$ and the Stokes force $T_C=3\pi d\nu_0\eta$, directed upwards. If it is assumed that the movement of the ball from point A to point B is uniform and the speed of the ball ν_0 is constant, the equation of Newton's second law in projection onto the vertical axis (D.1), (D.2) can be written as:

$$F_A - F_T + T_C = 0. \tag{7.1}$$

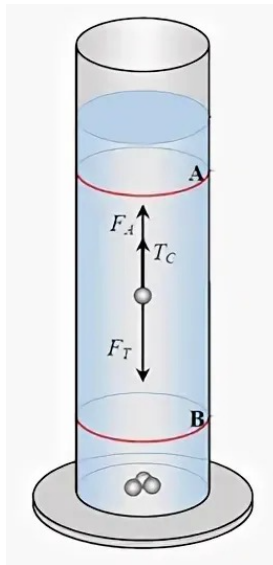


Fig. 7.1. Experimental setup

Substituting the expressions for the forces F_A , F_T and T_C , and the volume of the ball yields an expression for the coefficient of internal friction of the liquid:

$$\eta = \frac{1}{18} \frac{(\rho_1 - \rho_2)gd}{v_0} \quad (7.2)$$

The distance between the liquid surface and the upper mark is chosen so that the ball's speed stabilises in this section and its motion becomes uniform.

The steady-state speed of the ball over the section L (AB) is calculated using the formula:

$$v_0 = L / t.$$

where t is the time taken for the ball to travel between the upper mark A and the lower mark B. The viscosity coefficient of the liquid is then determined using the formula:

$$\eta = \frac{1}{18} \frac{(\rho_1 - \rho_2)gd^2}{L} \quad (7.3)$$

Procedure.

1. Determine the scale division of the instruments.

$$h \text{ sec} = \dots (\text{s}), \quad h \text{ distance} = \dots (\text{m}).$$

2. Measure the distance travelled by the pellet, equal to the distance between the upper mark A and the lower mark B of the vessel, $L = \dots$ m.

3. Measure the diameter of the pellet d using a microscope, enter the data into Table 7.1.

4. After lowering each pellet into the vessel, measure the time t taken to travel the distance L between the upper and lower marks. Repeat the measurements for the other pellets and record the data in Table 7.1.

5. Calculate the viscosity coefficient for each experiment. Record the average value of the viscosity of the liquid, specifying the units of measurement.

6. Compare the experimentally determined viscosity coefficient of the liquid with the standard value from a reference source.

7. Calculate the direct errors in measuring the and the fall time of a single pellet.

8. Write down the result of direct measurements (Table 7.1), taking into account the error and indicating the units of measurement in the SI system.

$$L = L \pm \Delta L, t = t \pm \Delta t.$$

9. Calculate the error in determining the viscosity as an error in an indirect measurement (I.18) (Table 7.1).

10. Write a conclusion summarising the skills acquired during the experiment and explaining the reasons for the discrepancy between the experimental and standard values of the viscosity coefficient.

11. Prepare a report.

— State the objective of the exercise.

— Describe the method for determining viscosity.

— Complete Table 7.1 with the measured data and calculated values.

— Answer the questions.

Table 7.1

Experimental data for calculating the viscosity coefficient

N	Diameter of a pellet, d_i (m)	Error, ΔL (m)	Time of motion, t_i (s)	Error, Δt (s)	Viscosity η_l (P · s)	Error, $\Delta \eta_l$ (P · s)
1						
2						
3						
4						
5						

Questions.

1. What is the difference between external and internal friction? What is the mechanism of viscosity?

2. What physical quantities determine viscosity?

3. How does viscosity in liquids and gases change with temperature?
4. What is the mean free path and effective cross-section? How do their values change with temperature?
5. What is the relationship between the effective diameter and the radius of molecular action?
6. What physical quantities determine the Stokes force?
7. Does the speed of a ball in a liquid depend on its diameter and material?
8. How can the pressure acting on a ball in a liquid be determined?
9. What physical quantities determine the buoyant force?
10. Is it possible to select conditions under which a lead pellet will float on the surface of a liquid?
11. What process within a living organism is simulated by this experimental setup?
12. Can viscosity serve as a diagnostic parameter in medicine? What pathologies can arise from deviations from the norm?

Laboratory 8

Examination of harmonic oscillations with the oscilloscope

The **objective** of this laboratory is to measure the characteristics of harmonic oscillations — amplitude and frequency — using an oscilloscope and to study the vibration trajectory when two mutually perpendicular harmonic oscillations with multiple frequencies are combined (Lissajous figures).

Instruments and accessories:

1. G3-7A sound generator.
2. S1-114/I oscilloscope.

Theoretical framework.

Oscillation is the motion of a body characterised by a certain degree of repetition. A common type is harmonic oscillation, in which the physical quantity $x(t)$ varies with time according to the sine or cosine law:

$$\begin{aligned}x(t) &= A\sin(\omega_0 t + \varphi); \\x(t) &= A\cos(\omega_0 t + \varphi),\end{aligned}\tag{8.1}$$

where A is the oscillation amplitude, i. e. the maximum value of $x(t)$; the argument of the sine or cosine ($\omega_0 t + \varphi$) is called the current phase of the oscillations; φ is the initial phase of oscillations at time $t=0$; ω_0 is the cyclic or natural frequency of oscillations, i. e. the number of oscillations per 2π ($\omega_0 = \frac{2\pi}{T}$), where T is the period of oscillations, or the time of one complete oscillation.

The simplest examples of harmonic oscillations include the oscillations of a mass attached to a horizontal spring, the motion of a mathematical pendulum and the orbital movement of an electron around a nucleus. The movements of bodies can be described as motion under the action of an elastic force proportional to the displacement of a material point from the equilibrium position in the form:

$$F = -kx, \quad (8.2)$$

where k is a constant.

The minus sign means that the elastic force, which plays the role of a restoring force, is always opposite in direction to the displacement x . The nature of the elastic force depends on the type of the oscillating system itself. Yet, the force is always directed toward the equilibrium position and opposite to the displacement. Newton's second law (D.1) for the system is written in scalar form to describe the motion of the load:

$$m \frac{d^2x}{dt^2} = -kx.$$

Transforming this equation yields:

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0, \quad (8.3)$$

where

$$\sqrt{\frac{k}{m}} = \omega_0. \quad (8.4)$$

The latter equation is the natural oscillation frequency determined by the parameters of the oscillatory system, k and m (8.4). Thus, expression (8.3) will take the form:

$$\frac{d^2x}{dt^2} + \omega_0^2x = 0. \quad (8.5)$$

Equation (8.5) is called the differential equation of harmonic oscillations. If an equation of this form is obtained from any law for a certain quantity x , the time dependence of this quantity represents a harmonic oscillation. The solutions of this equation are:

$$x(t) = A \sin(\omega_0 t + \varphi) \text{ or } x(t) = A \cos(\omega_0 t + \varphi).$$

Harmonic oscillations are the simplest type. Let us now consider a more complex case, when a material point simultaneously undergoes two mutually perpendicular oscillations with the same

frequency. This motion can be represented as the sum of two mutually perpendicular oscillations. Let one oscillation occur along the x -axis and the other along the y -axis of the Cartesian coordinate system:

$$\begin{aligned}x &= A\cos(\omega t + \varphi_1); \\y &= B\cos(\omega t + \varphi_2).\end{aligned}\tag{8.6}$$

The equation of the point's trajectory is obtained by eliminating the time t from equations (8.6). Let us rewrite (8.6) in the following form:

$$\begin{aligned}\frac{x}{A} &= \cos\omega t\cos\varphi_1 - \sin\omega t\sin\varphi_1; \\ \frac{y}{B} &= \cos\omega t\cos\varphi_2 - \sin\omega t\sin\varphi_2.\end{aligned}\tag{8.7}$$

Let us multiply the obtained equations by $\cos\varphi_2$ and $\cos\varphi_1$, respectively, and subtract the second equation from the first to obtain:

$$\frac{x}{A}\cos\varphi_2 - \frac{y}{B}\cos\varphi_1 = \sin\omega t\sin(\varphi_2 - \varphi_1).\tag{8.8}$$

Multiplying equations (8.7) by $\sin\varphi_2$ and $\sin\varphi_1$, respectively, and subtracting the second equation from the first yields:

$$\frac{x}{A}\sin\varphi_2 - \frac{y}{B}\sin\varphi_1 = \cos\omega t\sin(\varphi_2 - \varphi_1).$$

Squaring and adding the last two equalities gives the trajectory equation:

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - 2\frac{xy}{AB}\cos(\varphi_2 - \varphi_1) = \sin^2(\varphi_2 - \varphi_1).\tag{8.9}$$

Equation (8.9) is, in general, the equation of an ellipse, whose axes are rotated relative to the x and y coordinates. The orientation of the ellipse depends on the phase difference of the component oscillations. Let us define the trajectory formula for some particular cases from (8.9).

1. The phase difference is $\varphi_2 - \varphi_1 = 0$.

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB} = 0 \quad \text{or} \quad \left(\frac{x}{A} - \frac{y}{B}\right)^2 = 0.$$

This yields the equation of a straight line:

$$y = \frac{B}{A}x.$$

The resulting motion is a harmonic oscillation along this straight line with frequency ω and amplitude equal to $\sqrt{A^2 + B^2}$.

2. Consider the addition of oscillations whose phases — φ_1 and φ_2 — differ by $\frac{\pi}{2}$

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1. \quad (8.10)$$

Equation (8.10) is the canonical form of the ellipse equation, where the coordinate axes coincide with the ellipse axes. When the amplitudes of the oscillation components are equal ($A=B$), the ellipse degenerates into a circle.

Thus, when a material point undergoes two mutually perpendicular oscillations of the same frequency, its trajectory forms an ellipse. In some special cases, the ellipse may degenerate into a straight line or a circle.

If the frequencies of mutually perpendicular oscillations differ, being integer multiples of each other, the trajectory of the resulting movement takes the form of rather complex curves, known as Lissajous figures (Fig. 8.1).

For example, if the ratio of frequencies is $\frac{\omega_x}{\omega_y} = \frac{1}{2}$ and the phase difference is $\frac{\pi}{2}$, the oscillation equations have the form:

$$x = A\cos\omega t, y = B\cos(2\omega t + \frac{\pi}{2}).$$

While along the x -axis a point moves from one extreme position to the other, along the y -axis it departs from zero, reaches one extreme, then the other, and finally returns to zero.

Lissajous figures are conveniently observed on the oscilloscope screen when sinusoidal signals are applied to the horizontal and vertical inputs. Changing the amplitudes, frequencies and phases of these signals can yield various Lissajous curves (see Fig. 8.1).

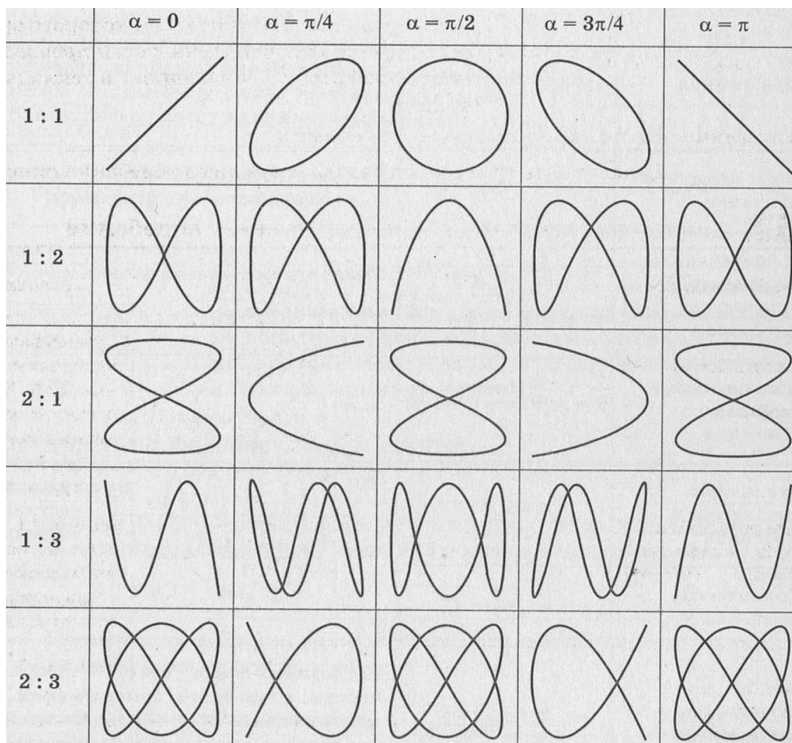


Fig. 8.1. Lissajous figures. The horizontal axis represents the phase difference α , and the vertical axis represents the frequency ratio ω_2/ω_1

Experimental part. Laboratory exercise steps.

The laboratory setup for the study of harmonic oscillations consists of a G3-7A sound generator and an S1-114/I oscilloscope.

The sound generator G3-7A produces harmonic oscillations in the frequency range $20\text{--}200 \cdot 10^3$ Hz. The oscilloscope S1-114/I, depending on the selected mode, is used to observe the oscillation shape, measure their frequencies and amplitudes or display Lissajous figures.

Procedure.

Exercise 1.

1. Assemble the equipment and connect both the generator and the oscilloscope to the power supply.
2. Apply harmonic voltage from the generator,

$$U = A(\omega_g t + \varphi),$$

to the oscilloscope's vertical deflection input of channel B, where ω_g is the frequency set with the sound generator dial. Adjust the generator knobs labelled 'reg. output' to set the specified frequency and amplitude A (use voltmeter).

3. Set the trigger type switches on the oscilloscope to 'auto' and 'internal'.

4. Obtain a stable image of the signal on the entire screen of the oscilloscope. The vertical size of the image is adjusted with the 'V/div' knob.

5. Select the sweep time with the 'time/div' switch so that 3—4 periods of the harmonic signal are visible on the oscilloscope screen. Achieve signal stability by smoothly synchronising the sweep frequency with the frequency of the signal under study using the 'level' and 'time/div' switches.

6. Position the resulting signal image symmetrically about the x -axis by moving it vertically using the '↑' handle.

Measure the characteristics of harmonic oscillations such as frequency and amplitude using an oscilloscope.

1. Calculate the amplitude A of the signal under study using the formula $A = \frac{l}{2} \cdot V/\text{div}$. To do this, measure the vertical size of the signal l , where l is the number of vertical divisions of the oscilloscope grid spanned by the entire signal. Take into account the setting of the 'V/div' switch, which should be in the extreme right position of the 'V/div — smoothly' knob. Record the measurement results in Table 8.1.

Table 8.1

Results of measurements for amplitude calculation

N	I (division)	V (division)	A(V)
1			
2			
3			

2. Calculate the frequency of the signal under study. To do this, measure the number of horizontal divisions l_1 per one oscillation period. Calculate the period of oscillation.

$$T = l_1 \tau_p,$$

given the position of the sweep time switch ‘time/div’ τ_p . Using the formula, $\nu = \frac{1}{T}$ determine the frequency of the signal under study.

The accuracy of measuring the period T of the signal can be improved by obtaining the number of horizontal divisions l_2 corresponding to several periods of oscillation, n . In this case $T = \frac{l_2 \tau_p}{n}$.

Record the results in Table 8.2.

Table 8.2

Measurement results for calculating frequency and period

N	τ_p	l_1	T	ν	n	l_2	T	ν	V_{gen}
1									
2									
3									
4									
5									

3. Measure the amplitudes and frequencies of five different harmonic voltages using the procedure described above. Compare the measurement results with the generator readings.

Exercise 2. Studying the trajectory of vibrations resulting from the addition of two mutually perpendicular harmonic oscillations (Lissajous figures).

To combine two mutually perpendicular oscillations with multiple frequencies, an oscilloscope is used. The device must have two pairs of mutually perpendicular deflecting plates, X and Y, in the cathode-ray tube and two amplifiers with outputs connected to these plates.

1. Apply voltage with a frequency of 50 Hz from the generator to the plates X. To do this, set the switch regulating the type of synchronisation to the ‘network’ position and set ‘time / div’ to ‘O–X’.

2. Using Table 8.1 and the values of the frequency ratio ω_2/ω_1 , calculate the frequency ω_1 , taking into account that ω_2 has a constant value of 50 Hz. For example:

$$\frac{\omega_2}{\omega_1} = \frac{2}{1} \rightarrow \frac{2}{1} = \frac{50}{\omega_1} \rightarrow \omega_1 = 25 \text{ Hz.}$$

3. By setting frequency multiples of 50 Hz on the sound generator using the dial, obtain stable images of Lissajous figures and sketch them. Explain the observed images and compare the frequencies of ν_{gen} with ν_{calc} .

4. **Prepare a report** covering the following required sections:

- The principal objective of the exercise.
- Description of research methods and equipment.
- Theoretical framework. Sketches of Lissajous figures presented in the description; equations of oscillations leading to the Lissajous figures.
- Procedure, experimental data (Tables 8.1 and 8.2) and data analysis.
- Conclusions. Explanation of the obtained images; comparison of the frequencies ν_{gen} and ν_{calc} .

Questions.

1. Which oscillations are called harmonic, and what are examples of oscillatory systems in everyday life?

2. List and explain characteristics of harmonic oscillations.
3. What movement does a point make if it simultaneously participates in two mutually perpendicular harmonic oscillations with the same frequencies?
4. Under what conditions will the trajectory of motion be a straight line and under what conditions will it be a circle?
5. What are Lissajous figures, and how can they be obtained experimentally? How can the ratio of the frequencies of the combined oscillations be determined from the appearance of a Lissajous figure?
6. How does a cathode-ray tube function?
7. List the functions of the main blocks in the oscilloscope block diagram.
8. What Lissajous figures appear on the screen when the following signals are applied to the X and Y inputs of the oscilloscope:

$$U_x = A\sin(\omega t);$$
$$U_y = A\cos(3\omega t).$$

9. How can the study of vibrational characteristics be applied in medical equipment?

Laboratory 9

Determining the focal length of a converging convex lens

The objectives of the exercise are to study image formation with lenses and explore the characteristics of a collecting lens. Determine the focal length of a spherical convex lens using two methods:

1. Lens formula.
2. Lens displacement.

Compare the results obtained by both methods.

Instruments and accessories:

1. Light source.
2. Screen with a slit.
3. Matte screen.
4. Collecting lens.
5. Diverging lens.
6. Optical bench.
7. Measuring rulers.

Theoretical framework.

Optical systems such as lenses, complex lenses are characterised by a number of parameters.

The **optical centre** of a lens is called point C , through which a light ray passes without changing its direction.

The **main optical axis** NN' is a straight line passing through the **centres of curvature** O and O' of the surfaces that bind a lens or a system of lenses (a centred system).

The **main focus** F is determined by a beam of rays entering the system from point F and leaving the system as a beam parallel to the main optical axis (Fig. 9.1).

Therefore, if the ray KL passes through the focus, the ray $K'L'$ leaving the system is parallel to the main optical axis. Rays AL and

BM , parallel to the main optical axis, can be considered as coming from an infinitely distant point located on the optical axis on the left. After passing through the system, they gather at point F' , which lies on the main optical axis. Point F' is the **second main focus** of the system. The planes drawn through the main foci F and F' perpendicular to the main optical axis NN' are called the **focal planes of the system**.

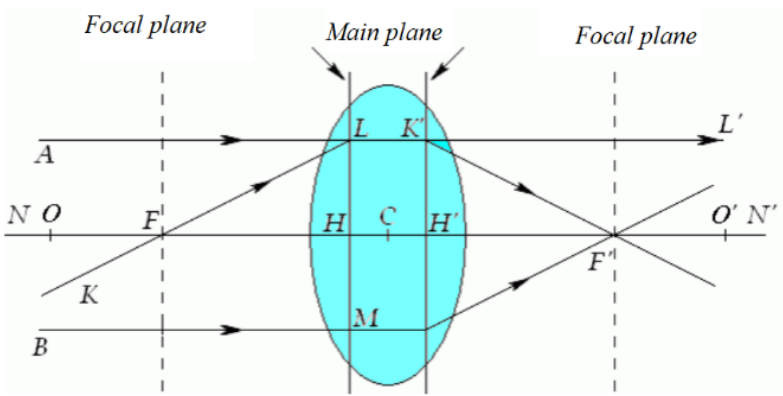


Fig. 9.1. Ray path in a lens

The principal planes are parallel to each other and to the focal planes of the system. The intersection points H and H' of the main planes of the system containing the main optical axis are called the main points of the system.

The distances from principal points to principal foci are called **focal lengths** (FH and $F'H'$). These distances are also referred to as F .

Thin lenses.

The simplest optical systems are thin lenses. A lens is considered thin if its thickness can be neglected in comparison with the radii of curvature of its bounding surfaces and its focal lengths.

For a thin lens, the focal length is measured from the optical centre. A thin lens has a single principal plane, common to both of its surfaces, which passes through the optical centre and is perpendicular to the main optical axis. The focal length of a thin lens is related to the radii of curvature limiting its spherical surfaces R_1 and R_2 by the following equation:

$$F = \frac{R_1 R_2}{(n-1)(R_1 + R_2)},$$

where $n = n_2/n_1$ is the relative refractive index of the lens material: n_2 stands for the refractive index of the lens material and n_1 for the refractive index of the environment.

It should be noted that in all formulas, distances measured to the right of the lens centre are taken as positive (+), while those measured to the left are taken as negative (-).

As a rule, the light source is placed on the left.

To construct the image of any point of an object, at least two rays emanating from that point should be chosen, whose directions after passing through the lens are precisely known.

The point where rays or their extensions intersect after passing through the lens will serve as an image of the object's point.

The following can be used as such rays:

1. Rays passing through the optical centre of the lens, which do not change their direction.
2. Rays passing through the main focus of the lens, which become parallel to the main optical axis after refraction.
3. Rays parallel to the main optical axis, which go through its main focus after passing through the lens.

Figure 9.2 shows examples of imaging in collecting (*a*) and diverging (*b*) lenses.

The distances from the optical centre of the lens to the object a , to the image b and the focal length of the lens F are related by the thin lens formula:

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{F}. \quad (9.1)$$

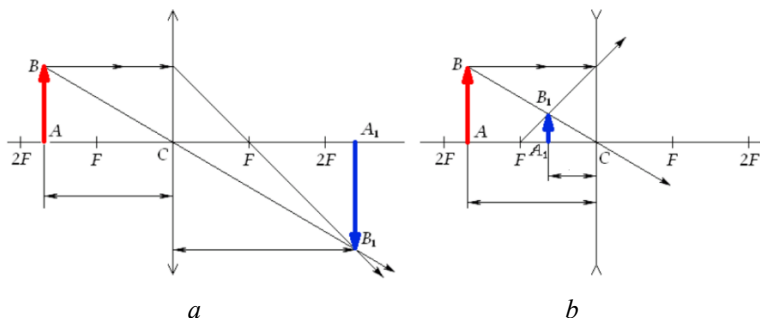


Fig. 9.2. The path of rays in collecting (a) and diverging (b) lenses with real (a) and virtual (b) images formed

The optical power of the lens D is inversely proportional to the focal length F :

$$D = \frac{1}{F}. \quad (9.2)$$

This experiment employs two techniques to obtain the focal length of a spherical lens: the lens formula method and the lens replacement method.

The **first method** requires determining the focal length of the lens using the lens formula. To find the focal length, a clear image of the object should be obtained and the distances to the object and the image measured.

The lens formula is written as follows:

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{F}.$$

It expresses the relationship between the object distance a , the image distance b and the focal length of a spherical lens F .

Therefore, several pairs of object and image distances are recorded to determine the focal length of the given lens using the formula above.

The **second technique**, called the lens displacement method, involves moving the lamp housing and the screen apart by a distance s . At this distance, there are two positions of the lens that form an image on the screen due to the reversibility of light. The focal length F is determined using the following formula:

$$F = \frac{s^2 - d^2}{4s},$$

where d is the distance between the two lens positions.

Experimental part. Laboratory steps.

Exercises 1.

First, apply **method 1**. Place the screen at a sufficiently large distance from the object (the letter F). Position the lens between them and move it until a clear image of the letter F is obtained.

Using the scale along the optical bench, record the positions of the lens, screen and object. Then move the screen to another position, adjust the lens until a clear image appears, and record the positions of all components. Repeat this procedure five times. Enter the data in Table 9.1. In this way, measurements are obtained for both a magnified image and a reduced image of the object. For each measurement, report the distance to the object, a , and its image, b . Repeat the measurement with the object distance (a) set at approximately $2F$ and at distances greater than $2F$. Use the lens formula (9.1) to calculate the focal length of each data pairing (Table 9.1). Finally, determine the mean focal length.

Table 9.1

Experimental data for calculating focal length. Method 1

N, i	a, mm	b, mm	F, mm	F_{mean}, m	$D,$ <i>dioptries</i>	k
1						
2						
3						

The magnification of a lens can be determined in two ways using the following relationships:

$$k = \frac{b}{a} = \frac{H}{h}, \quad (9.3)$$

where H and h are the linear dimensions of the image and the object respectively.

Exercise 2.

The focal length can be determined from the sizes of the object and its image, and from the distance of the image to the lens. Position the lens between the screen and the object so that the scaled screen produces an enlarged, clear image of the object. Use a ruler to measure the size of the image on the screen, L . The dimensions of the object are given in Figure 9.3.

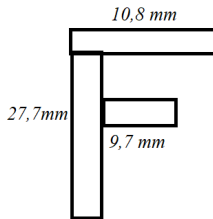


Fig. 9.3. Dimensions of the object

Measure the distance from the image to lens, b . Calculate the focal length of the lens using the formula.

The focal length of a lens can be determined in another way, using the ratio:

$$\frac{h}{H} = \frac{F}{b-F}. \quad (9.4)$$

Measuring the linear dimensions of the object and the image yields:

$$F = b \frac{h}{h+H}. \quad (9.5)$$

Repeat measurement three times. Enter the data in Tables 9.2 and 9.3.

Table 9.2

Experimental data for calculating focal length. Method 2

N	H (object size)	h (image size)	b (distance from the image to the lens)	F , m
1				
2				
3				

Table 9.3

Object and image position relative to the lens

Object position relative to lens	Image characteristics					
	Reality		Size		Orientation	
	Actual image	Virtual image	Increased	Reduced size	Direct	Reverse
1. $d > 2F$						
2. $d = 2F$						
3. $F < d < 2F$						
4. $d = F$						
5. $d < F$						
6. Biconcave thin lens (any position of the object)						

Prepare a report covering the following required sections:

- Formulas for calculating the focal length.
- Experimental design.
- Data Tables 9.1 and 9.2 with calculated values.
- Measurement error calculated.
- Conclusion. Answers to the questions and problems.

Questions and problems.

1. What is the principal optical axis of a lens?
2. Which points are called the principal foci, principal points and the optical centre of a lens?
3. What planes are called main planes and focal planes?
4. How are images of objects constructed using thin lenses?
5. List methods for determining the focal lengths of converging lenses.
6. How can the focal length of a converging or diverging lens be determined?
7. What is the linear magnification of a lens?
8. A thin lens has a focal length of $F=12$ cm. An object of height $h=5$ cm is placed at distances 1) $a_1=26$ cm and 2) $a_2=8$ cm. Calculate the distance b at which the image will be observed and determine the magnification k of the object. Refer to Table 9.3.
9. How does the optical power of a lens change when it is placed in a different medium?
10. Construct the image of an object as formed by a microscope.
11. Describe the structure of the eye. What organs comprise the light-conducting apparatus?
12. What is the functional significance of the cornea, pupil, lens and retina?
13. What is accommodation?
14. How is a colour image formed? Define colour blindness, hemeralopia, nyctalopia and photokeratitis?
15. List common visual aberrations.
16. How are myopia and hyperopia corrected?
17. How can visual impairment be prevented?

Laboratory 10

Experimental verification of Malus's law

The **objectives** of this exercise are to examine how light intensity depends on the angle between polaroids and to experimentally verify Malus's law.

Instruments and accessories:

1. A set of equipment consisting of a polariser and an analyser, with the angle between them adjustable using a goniometer.
2. Photocell.
3. Light source.
4. Microammeter.

Theoretical framework.

Polarisation of light is a property of light characterised by the ordered spatiotemporal orientation of the electric and magnetic field strength vectors of a light wave in a plane perpendicular to the light beam. The plane of polarisation passes through an electric vector (E-vector) and the direction of the light wave propagation. Such a wave, emitted by a single atom, is a plane-polarised electromagnetic wave. Natural light — sunlight or that coming from incandescent lamps — is not polarised, since it is the radiation of electromagnetic waves from many atoms. Each atom emits spontaneously at any moment, resulting in a set of electromagnetic waves with E-vectors oriented in all possible planes, where all directions of oscillation of the electric and magnetic vectors are equally probable. In plane-polarised light, E-vectors are oriented in one specific plane. If an E-vector changes in magnitude, polarisation is linear; if the change occurs in direction, polarisation is circular. If an E-vector changes both in magnitude and direction, polarisation is elliptical. Polarisation occurs when light is reflected and refracted at the boundary of two isotropic dielectrics. If the incident light is natu-

ral, the refracted and reflected rays are partially polarised. The degree of polarisation depends on the angle of incidence α and the refractive index of the second medium relative to the first: $n_{21} = n_2/n_1$.

Devices called polarisers generate polarised light. Polarisers transmit light rays with a single direction of oscillation of the electric vector E to a specific plane called the main plane of the polariser. This plane passes through the optical axis of the polariser and the incident beam. At the output of the polariser, the light is plane (linearly) polarised. The intensity of natural light decreases by half after passing through the polariser

$$I_2 = I_1/2. \quad (10.1)$$

In isotropic media, the physical properties of a substance are independent of direction. Anisotropy is the dependence of mechanical, electrical and optical properties on direction: in an anisotropic medium, the physical properties are not the same in different directions, i. e. the properties change from point to point. Anisotropic media can split an incident light beam into two beams — ordinary and extraordinary — which propagate in different directions at different phase velocities and are polarised in mutually perpendicular planes. This is the phenomenon of double refraction, observed when light strikes anisotropic crystals such as quartz, tourmaline or Iceland spar. In double refraction, the law of refraction applies to the ordinary beam but not to the extraordinary beam. The oscillations of the ordinary beam are perpendicular to the main plane of the crystal, while the oscillations of the extraordinary beam lie in the main plane. The optical axis of the crystal is the direction along which double refraction is absent.

When polarised light passes through optically active substances, the plane of light polarisation rotates around the direction of the beam. The OAB rotates the plane of polarisation by an angle φ , which is proportional to the path L travelled by the light in the optically active substance.

$$\varphi = \alpha L, \quad (10.2)$$

where α is the rotation constant determined by the characteristics of the substance: its temperature and wavelength. This property is

used in some medical and biological studies. In particular, polarimeters are used in saccharimeters — optical devices designed to measure the concentration of optically active solutions. These instruments measure the angle of rotation φ of the plane of polarisation.

Polarisation devices analyse light. An analyser is a polariser that is used to study pre-polarised light. When the planes of polarisation of the polariser and analyser are rotated, the light intensity changes according to **Malus's law**:

The intensity of light emerging from the analyser is proportional to the square of the cosine of the angle between the main planes of the polariser and analyser:

$$I = I_0 \cos^2 \varphi. \quad (10.3)$$

Light polarisation can be observed and Malus's law verified using a device consisting of a light source, two polaroids, a photocell and a microammeter. One of the polaroids, mounted on a goniometer and rotating around the direction of beam propagation, serves as the polariser. The second polaroid, which is fixed, serves as the analyser. A photocell is installed behind the analyser, included in the microammeter circuit. The photocurrent is directly proportional to the intensity of light falling on the photocell. Therefore, the intensity of light can be estimated from the readings of the microammeter. The degree of light polarisation equals the intensity of partially polarised light transmitted through the analyser, while the intensity distribution follows a normal Gaussian distribution. The experimental curve obtained is compared with the graph constructed from theoretical calculations; if the graphs coincide, **Malus's law** is confirmed.

Experimental part. Laboratory exercise steps.

1. Find the division value of the microammeter.
2. To study the angular dependence of the light intensity I transmitted through two polaroids, rotate the goniometer clockwise, recording the microammeter readings every 10° from 180° to 360° . Record the measurement results, I_l , in Table 10.1.

Table 10.1

Experimental data for testing Malus's law

Polariser rotation angle, α (degree)	Angle between polaroids, φ (degree)	$\cos^2 \varphi$	I_1 (μA)	I_2 (μA)	I_{mean} (μA)	$I_{max} \cdot \cos^2 \varphi$
180	+90					
190	+80					
200	+70					
210	+60					
220	+50					
230	+40					
240	+30					
250	+20					
260	+10					
270	0					
280	-10					
290	-20					
300	-30					
310	-40					
320	-50					
330	-60					
340	-70					
350	-80					
360	-90					

3. Repeat the experiment, rotating the polariser counterclockwise. Record the measurement results, I_2 , in Table 10.1.

4. Calculate $\cos^2 \varphi$ and the mean of intensity I_{mean} ; record the data in Table 10.1.

5. Based on the experimental results, find in the table the maximum intensity I_{max} , which should approximately correspond to the angle $\varphi = 0^\circ$ between the polaroids.

6. Calculate the values of $I_{max} \cdot \cos^2 \varphi$ (Table 10.1).

7. Plot a graph of the dependence of light intensity (I_{mean} , μA) on the angle between the polaroids (experimental curve). The graph is plotted on graph paper, with a pencil; the axes must be labelled and the unit segments indicated. A smooth curve is drawn to pass as closely as possible to the graph points (Introduction, section 5).

8. On the same axes, plot a theoretical curve of the light intensity distribution $I_{\text{max}} \cos^2 \varphi$ from the angle between the polaroids.

9. Formulate a conclusion that summarises the skills acquired during the experiment and evaluates how well the experimental graph corresponds to the intensity distribution of polarised light predicted by **Malus's law** (10.3).

10. **Prepare a report** covering the following required sections:

- The objective of the experiment.
- Experimental design.
- Table 1 containing the experimental data and calculated values.
- A constructed graph of $I_{\text{max}} \cos^2 \varphi$ showing the comparison between the theoretical curve and the experimental data.
- Conclusion on the agreement of the experimental results with Malus's law.

11. **Answers to the questions and tasks**

Questions and tasks.

1. What is the physical nature of light? What is the difference between natural and polarised light?
2. What is polarisation? What types of light polarisation exist?
3. What methods are used to obtain polarised light?
4. What does Malus's law state?
5. What device is used to measure light intensity?
6. What do the law of reflection and the law of refraction of light state? What characterises the refractive index?
7. What is total internal reflection? What determines Brewster's angle?
8. What is double refraction?

9. What is the difference between isotropic and anisotropic media?
10. What are optically active substances? On which property of OAS are polarimetry and saccharimetry based?
11. What is the operating principle of a polarising microscope?
12. What is photoelasticity?
13. List the medical techniques that utilise polarisation.

Laboratory 11

Study of light diffraction

The **objective** of this exercise is to become familiar with the method of observing a spectrum using a diffraction grating and to study the methodology for determining the characteristics of the spectrum.

Instruments and accessories:

1. Installation containing a diffraction grating, a telescope, and a collimator.
2. Micrometer.
3. Mercury lamp.
4. Transformer.
5. Ammeter.

Theoretical framework.

Light diffraction is the phenomenon in which light deviates from a straight-line path in a medium with sharp inhomogeneities. It occurs when light passes along the edge of a sharp boundary between light and shadow. A **diffraction grating** is a system of periodically repeating parallel slits. The simplest diffraction grating is an optical device consisting of a glass plate with a large number of regularly repeating parallel lines (up to 10,000 lines per millimetre) etched on it, or a set of slits spaced equally apart. The lines act as opaque obstacles in the path of the light wave, allowing only part of the light beam to reach the observation direction. A diffraction grating splits non-monochromatic light into a diffraction spectrum and functions as a dispersion device that enables spectral analysis of light (Fig. 11.1).

As Figure 11.1 shows, a diffraction grating makes it possible to study the spectral composition of the incident radiation. The main characteristic of a diffraction grating, which determines its optical properties, is the grating constant (period):

$$d = 1/N = a + b, \quad (11.1)$$

where N is the number of lines, a is the slit width, and b is the line width.

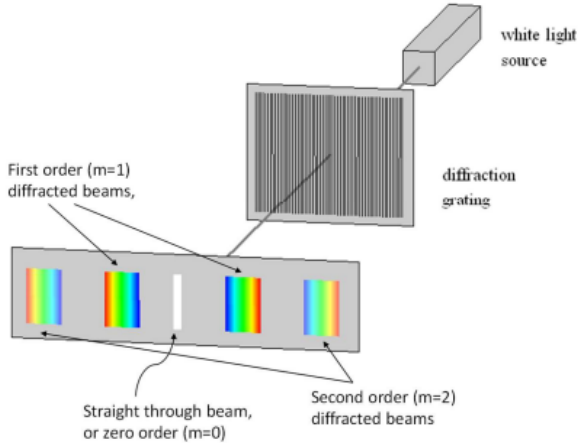


Fig. 11.1. Setup for observing light diffraction using a diffraction grating

The optical path difference of waves coming from two adjacent slits equals:

$$\delta = d \sin \varphi. \quad (11.2)$$

The maxima of the diffraction pattern intensity are observed for a diffraction grating under the condition:

$$d \sin \varphi = k \lambda, \quad (11.3)$$

where $k = \pm 0, 1, 2, \dots$ is the order of the spectrum (main maxima).

The wavelength of the source can be determined by the position of the diffraction maxima.

Additional minima are formed between the main maxima. The number of these minima is $N-1$. The number of additional maxima is $N-2$. The intensity of the main maxima decreases as the ordinal number k increases. The greater the number of slits on a grating,

the higher the intensity of the principal maxima and the narrower the spectral lines. If the grating is illuminated with monochromatic light, the maxima will appear in that colour. If illuminated with white light, all the principal maxima will decompose into a colour spectrum from violet to red, except for the central maximum ($k=0$), which remains white (Fig. 11.1).

A double arrangement of the maxima is observed for a source containing two wavelengths. The resolving power (resolution) of a diffraction grating characterises the ability of the grating to produce a separate image of two close spectral lines.

$$R = kN = \frac{\lambda}{\Delta\lambda}. \quad (11.4)$$

N is the number of grating slots, $\Delta\lambda$ is the minimum difference in wavelengths of two monochromatic lines of equal intensity distinguished in a spectrum.

According to the Rayleigh criterion, if the maximum of one wavelength coincides with the minimum of a second wavelength, the maxima can be considered disjoint. The Rayleigh resolution characterises the image quality of spectral instruments used for closely spaced points or lines, as well as optical instruments designed for the eye.

Dispersion is determined by the ratio of the angular distance between two spectral lines to the difference in their wavelengths:

$$D = \frac{\Delta\varphi}{\Delta\lambda}. \quad (11.5)$$

If the measurement is made with an ocular micrometer, dispersion can be determined as the ratio of the linear distance between spectral lines to the difference in their wavelengths:

$$D = \frac{\Delta S}{\Delta\lambda}. \quad (11.6)$$

The angular dispersion for a diffraction grating is given by the equation:

$$D = \frac{k}{d \cos\varphi}. \quad (11.7)$$

For small deflection angles, the dispersion of the grating is constant, so the diffraction spectra are called normal, in contrast to the spectra obtained using glass prisms. For prisms, the angular dispersion in the red part of the spectrum is less than in the violet. The use of a diffraction grating is the basis for many types of spectral analysis. Knowing the diffraction grating constant, one can find the wavelength of the incident light (direct problem)

$$\lambda = \frac{d(S-S_1)}{2kf}, \quad (11.8)$$

where $k=1; 2; 3$ is the spectrum order; $f=939$ mm is the focal length of the telescope lens; S and S_1 are the readings of the line positions on the right and left, respectively.

Knowing the wavelength of the incident light (for example, $\lambda_{\text{green}}=546$ nm; $\lambda_{\text{yellow}}=577$ nm), the grating constant can be determined as follows (inverse problem):

$$d = \frac{2k\lambda f}{(S-S_1)}. \quad (11.9)$$

Experimental part. Laboratory exercise steps.

1. Using an autotransformer, apply a voltage to the lamp so that a current of approximately 2 A flows through it. In a steady-state mode, after 3—5 minutes, the current should stabilise at 1.2 A. If the voltage drops, the lamp should be turned off temporarily.

2. Observe through the eyepiece and locate the zero (central) white maximum.

3. Turn the screw smoothly to move the crosshair in the eyepiece to the right until the first-order spectrum appears. Align the crosshair with the violet line of the first order and measure its position on the micrometer scale.

4. Align the crosshair with the green and yellow lines in succession. Repeat the measurements for the second and third orders of the spectrum, and record the data in Table 11.1.

5. Return the crosshair to the central maximum and, moving it to the left, repeat the experiment for the lines of the first, second, and third order on the left.

6. Calculate the diffraction grating period for the green and yellow lines of the first, second and third order using formula (11.9). Use the SI system in the calculations.

7. Find the average value of the grating period d .

8. Knowing the grating period, determine the wavelength of the violet lines for the first, second and third order using formula (11.8).

9. Write down the conclusion about the possibilities of using this method and the skills acquired while performing the experiment.

10. Prepare a report covering the following required sections:

— The objective of the exercise.

— The main formulas for diffraction on a grating and the formula for the resolving power of the grating.

— Experimental design.

— Table 11.1 completed with experimental data and the calculated value of the grating constant.

— Conclusion on the applicability of this method.

— Answers to the questions and tasks.

Table 11.1

**Experimental data for calculating
the diffraction grating period**

Order of maximum k	Right count S	Left count S_l	Lattice period d (m)	Wavelength λ (nm)
1 Purple Green Yellow				— 546 577
2 Purple Green Yellow				— 546 577
3 Purple Green Yellow				— 546 577

Questions and tasks.

1. What is light, and what are its properties?
2. What are wavelength, speed of light and frequency? How are these quantities related to the properties of the medium? What formulas connect these quantities?
3. What is a monochromatic wave? What wavelengths are included in the spectrum of white light?
4. What is wave-particle duality?
5. What is a diffraction grating, and what is the order of the grating?
6. What is diffraction? What is the fundamental difference between Fresnel and Fraunhofer diffraction?
7. When light diffracts on a grating, under what conditions are the intensity maxima of a diffraction pattern observed?
8. Draw a diagram showing the dependence of light intensity on the diffraction angle in a grating.
9. What is resolving power?
10. What is the Rayleigh criterion?
11. How is the dispersion of a diffraction grating determined?
12. What is spectral analysis?
13. Provide examples of biological and medical research methods based on spectral analysis.

Laboratory 12

Determining the wavelength using a Fresnel biprism

The **objective** of this exercise is to study the phenomenon of interference and become familiar with the method of measuring the wavelength of light using interference devices.

Instruments and accessories:

1. An installation for obtaining an interference pattern, consisting of a Fresnel biprism.
2. Young' slits and an ocular screw micrometer.
3. A light source, light filters and a collecting lens, mounted on an optical bench on mobile carriages.
4. A measuring ruler.

Theoretical framework.

Interference is the redistribution of light intensity in space caused by the superposition of electromagnetic waves. An important condition for observing interference is the coherence of waves.

Oscillations or waves of the same frequency that maintain a constant phase difference over time are called **coherent**. When coherent waves are superimposed, the light flux is redistributed in space, concentrating wave energy in certain regions to form intensity maxima, while in other regions the waves cancel each other, forming intensity minima.

A stable pattern of distribution of areas of maximum and minimum intensity is called an **interference pattern**. Patterns in the intensity distribution — stripes or circles — are determined by the arrangement and elements used to produce interference. All natural sources emit light spontaneously and chaotically, so waves from different sources do not interfere. Various methods exist for pro-

ducing coherent waves. One of these methods was proposed in 1802 by Thomas Young. The light source was a slit, from which the wave passed through two narrow openings in the screen. Such slits act as coherent light sources, and interference is observed in the region where the waves from them overlap (the interference field).

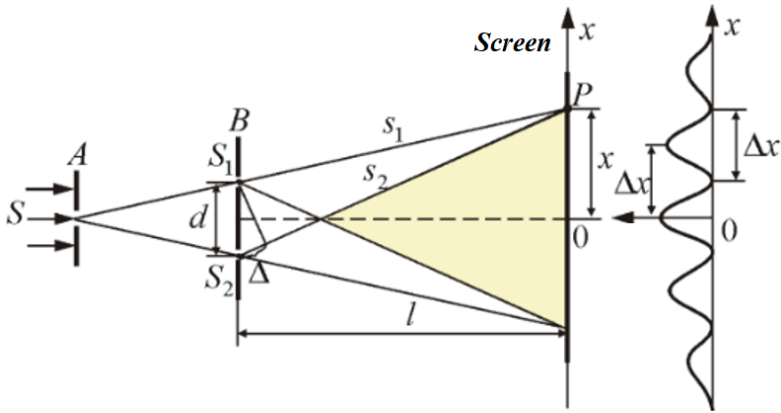


Fig. 12.1. Young's method for obtaining coherent waves

Fresnel biprism interference observation is another method of monitoring the phenomenon and is used to determine the wavelength of light. A biprism is two prisms with small refractive angles, joined at their bases. After refraction in a biprism, a beam of light falling from a slit, which serves as a linear source, splits into two overlapping beams, as if emanating from two virtual images of the slit (Fig. 12.1). Since these sources are coherent, the light beams will interfere in the space behind the biprism. The resulting pattern will appear as alternating light and dark fringes, with their positions determined by the following condition:

$$(x_m)_{max} = \frac{m\lambda l}{a}, \quad (12.1)$$

$$x_m = (2m + 1) \frac{l}{2d} \lambda, \quad (12.2)$$

where λ is the wavelength, l is the distance from the source (slit) to the observation point (eyepiece), d is the distance between the imaginary sources, and m is the band number from the central maximum.

The distance between two adjacent — light or dark — bands is given by:

$$\lambda = \frac{d\Delta x}{l}, \quad (12.3)$$

where Δx is the width of one strip.

If the distance between the images D is measured with an ocular micrometer, along with the distance from the slit to the lens a and from the lens to the micrometer b , the distance between the sources can be determined according to the thin lens formula:

$$d = \frac{a}{b} D. \quad (12.4)$$

The expression for determining the wavelength is:

$$\lambda = \frac{aD}{b} \frac{\Delta x}{l}. \quad (12.5)$$

Experimental part. Laboratory exercise steps.

1. Switch on the light source and achieve a stable interference pattern in the eyepiece using a red filter.

To obtain a stable image, align the illuminator window, slit, biprism, and ocular micrometer at the same height. The clarity of the interference pattern is adjusted by varying the slit width using the screw. Red and green filters are used to create a monochromatic beam. Measurements of the distance between fringes are made using an ocular micrometer: whole numbers from the position of the mark on the micrometer scale, and tenths and hundredths from the micrometer drum. A collecting lens is installed to obtain images of the slits. For a clear image in the ocular micrometer, the lens position should be adjusted smoothly or fine-tuned as necessary.

2. Rotate the micrometer screw smoothly to align the crosshair with one of the bright lines at the centre of the visual field and read the position of line N_1 .

3. Move the crosshair down to the fifth strip from the initially selected one. Read N_2 .

4. Repeat measurements described in points 2 and 3 five times and record the results in Table 12.1.

Table 12.1

**Measurement results for the red wave for calculating
the distance between stripes**

Red wave						
i	Width of stripes		Number of strips n	Width of one strip Δx	$\overline{\Delta x}$	l, mm
	N_1	N_2				
1			5			
2						
3						
4						
5						

5. Use a measuring ruler to find the distance l .

6. Install a collecting lens on the optical bench and measure the positions of imaginary sources N'_1 and N'_2 , using a similar technique. Record the measurement results for the red filter in Table 12.1.

7. Use a measuring ruler to determine the distances a and b .

8. Carry out the experiment following points 2 to 7 using a green filter. Record the data in Tables 12.2 and 12.4.

9. Using Tables 12.2 and 12.4, calculate the width of one strip $\Delta x = \frac{N_2 - N_1}{n}$ (the distance between stripes 1 and 5 is divided by the number of stripes n) for red and green.

10. Using Tables 12.3 and 12.4, calculate the distance $D = N'_2 - N'_1$ for red and green.

Table 12.2

**Measurement results for the green wave
for calculating the distance between stripes**

Green wave						
i	width of stripes		Number of strips n	Width of one strip Δx	$\overline{\Delta x}$	l, mm
	N_1	N_2				
1			5			
2						
3						
4						
5						

Table 12.3

Measurement results for the red wave

Red wave						
i	N'_1	N'_2	D_i	\overline{D}	a, mm	b, mm
1						
2						
3						
4						
5						

Table 12.4

Measurement results for the green wave

Green wave						
i	N'_1	N'_2	D_i	\overline{D}	a, mm	b, mm
1						
2						
3						
4						
5						

11. Using formula 12.5, determine the wavelength of red and green. Use the average values of Δx and D .

12. Using a reference book, find the wavelengths of red and green light and compare them with the calculated values.

13. Repeat the experiment using Young's slits instead of a biprism. Note that the distance l is measured from the eyepiece to the slits. Otherwise, the measurement procedure and calculation formulas are the same as in the biprism experiment.

14. Formulate a conclusion evaluating the method studied, comparing the standard and experimental values of the red and green wavelengths, and listing the skills acquired during the experiment.

Questions and tasks.

1. What is interference?
2. What is an interference pattern?
3. What properties do coherent sources have? Why are natural sources incoherent?
4. What methods are employed to obtain coherent waves?
5. What is a monochromatic wave?
6. Describe the physical nature of radiation.
7. What is the range of visible light on the electromagnetic wave scale?
8. What is the optical path difference?
9. How does the interference pattern change if the width of the linear source is increased?
10. How will the interference fringes shift if the distance between the sources is increased or decreased?
11. How does the fringe width change if the wavelength is reduced by half?
12. Draw a diagram of a device with a Fresnel biprism.
13. In which medical techniques is the phenomenon of interference used?

III. SUPPORT MATERIALS

Greek alphabet

A	α	alpha
B	β	beta
Γ	γ	gamma
Δ	δ	delta
E	ϵ	epsilon
Z	ζ	zeta
H	η	eta
Θ	θ	theta
I	ι	iota
K	κ	kappa
Λ	λ	lambda
M	μ	mu
N	ν	nu
Ξ	ξ	xi
O	\omicron	omicron
Π	π	pi
P	ρ	rho
Σ	σ	sigma
T	τ	tau
Υ	υ	upsilon
Φ	ϕ	phi
X	χ	chi
Ψ	ψ	psi
Ω	ω	omega

Common prefixes in the metric system

Prefix	Abbreviation	Multiply by	Prefix	Abbreviation	Multiply by
tera	T	10^{12}	milli	m	10^{-3}
giga	G	10^9	micro	μ	10^{-6}
mega	M	10^6	nano	n	10^{-9}
kilo	k	10^3	pico	p	10^{-12}
deci	d	10^{-1}	femto	f	10^{-15}
canti	c	10^{-2}	atto	a	10^{-18}

Basic formulas

Mechanics	
<i>Translational (rectilinear) motion</i>	
<i>Uniform</i>	<i>Uniformly accelerated</i>
Path $S = vt$	$S = S_0 + v_0 t + \frac{at^2}{2}$ $S = \frac{v^2 - v_0^2}{2a}$ if $v_0 = 0 \Rightarrow S = \frac{at^2}{2}; S = \frac{v^2}{2a}$
Displacement (m) along the axis x $S_x = x - x_0 = v_x t$	
Coordinate (m) $x = x_0 + v_x t$	$x = x_0 + v_x t + \frac{at^2}{2}$
Path $l = S_1 + S_2 + S_3$	The path corresponds to the area under the graph. $S_{\text{Tp.}} = \frac{v_0 + v_t}{2} l$
Speed (m/s) $v = \frac{S}{t}$	$v = v_0 + at,$ if $v_0 = 0, v = at$
Average speed $v_0 = \frac{S_1 + S_2 + S_3}{t_1 + t_2 + t_3}$	
Velocity (m/s) $v_x = \frac{S_x}{t} = \frac{x - x_0}{t}$	
Acceleration (m/s ²) $a = 0$	$a = \frac{v - v_0}{t} \quad \text{or} \quad a = \frac{v^2 - v_0^2}{2S}$

Curvilinear motion (in a circle)		
Rotation angle $\varphi = \varphi_0 + \omega_0 t + \frac{\beta t^2}{2}$ β is angular acceleration	Angular velocity $\omega = \omega_0 + \beta t$ $\omega = 2\pi\nu = \frac{2\pi}{T}$ (rad/s) ν (Hz) is the frequency, T (s) is period	Centripetal acceleration (normal) $a_n = \frac{v^2}{R} = \omega^2 R$
path $l = \varphi R$ length circumference $l = 2\pi R$	Linear speed $v = \frac{l}{t} = \frac{2\pi R}{T}$	tangential acceleration $\vec{a}_\tau = [\vec{\beta} \times \vec{R}]$
relationship between linear and angular velocity $\vec{v} = [\vec{\omega} \times \vec{R}]$		full acceleration $\vec{a} = \vec{a}_\tau + \vec{a}_n$
First cosmic velocity: if $r \approx R_{\text{Earth}}$ $v = \sqrt{gr}$; if $r = R_{\text{Earth}} + h$ $v = \sqrt{G \frac{M_{\text{Earth}}}{(R_{\text{Earth}} + h)}}$		

Gravity $F_g = mg$ Mass m (kg) is the measure of body inertia. The force of gravitational attraction $F_g = G \frac{m_1 m_2}{R^2}$ $G = 6,67 \cdot 10^{-11} \left[\frac{N \cdot m^2}{kg^2} \right]$ Friction force $F_{fr} = -\mu N$ Elastic force $F_{el} = -k\Delta x$ (Hooke's law)	Momentum $\vec{p} = m\vec{v}$ (kg·m/s) Law of conservation of momentum. The vector sum of the momenta of all bodies for a closed system is constant. $\vec{p}_1 + \vec{p}_2 = const$ Elastic collision. The law of conservation of momentum and mechanical energy is satisfied: $m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_3 + m_2\vec{v}_4$ $E_1 + E_2 = E_3 + E_4$ Perfectly inelastic collision: $m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{u}$ <i>(The law of conservation of mechanical energy is not satisfied)</i>
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Mechanical work (J) $A = FS\cos\alpha = \Delta E$	Mechanical energy (J) Kinetic energy $E_k = \frac{mv^2}{2} \geq 0$
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<p>The work done along a closed path is zero.</p> <p>$A > 0$ if $\alpha = 0 \Rightarrow \cos\alpha = 1$</p> <p>$A < 0$ if $\alpha = 180^\circ \Rightarrow \cos\alpha = -1$</p> <p>$A = 0$ if $\alpha = 90^\circ \Rightarrow \cos\alpha = 0$</p> <p>Power (W)</p> $N = \frac{A}{t} = Fv$	<p>Potential energy</p> $E_p = mgh, E_p = \frac{k\Delta x^2}{2}$ <p>Law of conservation of total mechanical energy (for closed systems):</p> $E_m = (E_k + E_p) = \text{const}$
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<p>Newton's second law</p> $\vec{a} = \frac{\vec{F}}{m}, \vec{F} = \sum_{i=1}^n \vec{F}_i$ $\vec{F} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$ <p>Newton's third law $F_1 = -F_2$</p> <p>Torque $\vec{\tau} = \vec{F} \times \vec{r}$ (N·m)</p>	
<p>The condition for static equilibrium is that the net force and net torque are equal to zero.</p> $\vec{F} = \sum_{i=1}^n \vec{F}_i = 0, \vec{\tau} = \sum_{i=1}^n \vec{\tau}_i = 0$ <p>Equilibrium condition of the lever: $F_1 l_1 = F_2 l_2$ (l is shoulder)</p> <p>Golden rule of mechanics: $F_1 S_1 = F_2 S_2$</p> <p>Gain in the strength of the movable block: $F = \frac{P}{2}$</p> <p>Efficiency $\eta = A_{\text{useful work}} / A_{\text{expenses}} \times 100\%$</p> <p>Pressure $P = F/S$ (Pa) (S, m^2 is square)</p>	

Translational motion	Rotational motion
$m = \rho V$ (kg) F (N) $P = mV$ (kg · m/s) $F = ma$ $\sum_{i=1}^N \vec{F}_i = \frac{d\vec{P}}{dt}$ $W_k = \frac{mv^2}{2}$ $\sum_{i=1}^N P_i = \text{const}$ $A = FS \cos\alpha$	$I = mr^2$ (kg · m ²) $M = Fl$ (N · m) $L = Pl = I$ (kg · m ² /s) $M = I\beta$ $\sum_{i=1}^N M_i = \frac{dL}{dt}$ $W_k = \frac{I\omega^2}{2}$ $\sum_{i=1}^N I_i \omega_i = \text{const}$ $dA = Md\varphi$

Properties of solids and liquids	
Hooke's law for a rod: $\sigma = E\varepsilon$ <i>E</i> is Young's modulus (modulus of elasticity) Relative deformation: $\varepsilon = \frac{\Delta l}{l} = \frac{\sigma}{E}$ Material stress, shear stress: $\sigma = \frac{F_{\perp}}{S}$	
Linear expansion of solids: $l = l_0(1 + \alpha\Delta T)$ α is the coefficient of linear expansion: $\Delta T = T - T_0 = 273K$ Volumetric expansion of solids and liquids: $V = V_0(1 + \beta\Delta T)$ $\beta \approx 3\alpha$ is the coefficient of volumetric expansion. Density change: $\rho = \frac{\rho_0}{(1 + \beta\Delta T)}$	
Surface tension: $\sigma = \frac{F_{\text{т.н.}}}{l} = \frac{W_p}{S}$. Excess (Laplace) pressure: $P_{\text{ex.}} = \pm \frac{2\sigma}{r_0}$ + for concave meniscus, wetting liquid – for convex meniscus, non-wetting liquid Height of liquid rise in a capillary: $h = \frac{2\sigma}{\rho g R}$	
Pressure in a column of liquid or gas: $p = \rho gh$ $1 \text{ mmHg} = 133.3 \text{ Pa}$ $1 \text{ atm} = 1.013 \cdot 10^5 \text{ Pa} = 760 \text{ mmHg}$ $\text{Normal BP} = 760 \text{ mmHg} = 101300 \text{ Pa} = 1.013 \cdot 10^5 \text{ Pa}$ <i>For every 12 m of height P changes by 1 mmHg</i>	
Archimedes' force: $F_A = \rho_{\text{liquid}} g V_r$ Weight of a body in a liquid: $P' = P - F_A = mg - F_A$ Condition for floating bodies: $\rho_{\text{body}} \leq \rho_{\text{liquid}}$	

Hydrodynamics	
Compressibility	$\chi = \frac{-\Delta V}{V} \frac{1}{\Delta P}$
Continuity theorem: through a smaller cross-sectional area, the liquid flows faster	$Sv = \text{const}$
Bernoulli's equation: the sum of the dynamic, hydrostatic and static pressures in a steady flow remains constant	$\rho \frac{v^2}{2} + \rho gh + P = \text{const}$
Torricelli's formula determines the flow velocity of a jet	$v = \sqrt{2gH}$

<p>Poiseuille formula: Q is volumetric flow rate, $(p_1 - p_2)$ is pressure difference l is pipe length, R is radius</p>	$Q = \frac{dV}{dt} = \frac{\pi R^4 (P_1 - P_2)}{8l\eta}$
<p>Reynolds number: Kinematic viscosity Reynolds criterion $Re \approx 2300$ $Re > Re_{cr}$ is a turbulent flow $Re < Re_{cr}$ is a laminar flow</p>	$\Re = \frac{\rho v D}{\eta} = \frac{v D}{\nu}$ $\nu = \frac{\eta}{\rho}$
<p>Newton's equation (internal friction force): $\frac{dv}{dr}$ is velocity gradient (rate of change in velocity in the direction $r \perp S$) For Newtonian fluids, viscosity does not depend on the velocity gradient</p>	$F_{fr.} = \eta \frac{dv}{dr} S = \eta grad \bar{v}$
<p>Stokes' force: r is the radius of a spherical body, λ is mean free path $(\lambda \sim 1/P)$, $\sigma = \pi d^2$ is effective cross-section, and d is effective diameter</p>	$F_s = 6\pi\eta r v$ <p>(in liquid)</p> $F_s = \frac{1}{3} \rho \lambda v_{cp.}$ <p>(in gas)</p> $\lambda = \frac{1}{\sqrt{2} \pi d^2 n} = \frac{1}{\sqrt{2} \sigma n}$ $v_{mean.} = \sqrt{\frac{8RT}{\mu}}$
<p>Langevin equation (Newton's law for a liquid): the force consists of two parts — the average drag force $(-\beta \bar{v})$ and the rapidly fluctuating part $g(t)$</p>	$m \frac{d\bar{v}}{dt} = -\beta \bar{v} + \overline{g(t)}$ $\overline{F_{drag}} = -\beta \bar{v} = -6\pi r \bar{v}$
<p>Diffusion phenomenon. Fick's first law The flux density j is linearly proportional to the concentration gradient dC/dx. The constant D is called the diffusion coefficient</p>	$j_x = -D \frac{dC}{dx}$
<p>Einstein relation between diffusion and viscosity:</p>	$D = \frac{k_b T}{6\pi r}$

Mechanical vibrations and waves	
Period $T = \frac{1}{\nu} = \frac{t}{n} = \frac{2\pi}{\omega}$ T is the time required for one oscillation; n is the number of oscillations	Period of a mathematical pendulum: $T = 2\pi \sqrt{\frac{l}{g}}$ Period of a spring pendulum: $T = 2\pi \sqrt{\frac{m}{k}}$
Frequency $\nu = \frac{1}{T} = \frac{n}{t} \text{ (s}^{-1} = \text{Hz)}$	Frequency circular (cyclic) $\omega = 2\pi\nu = \frac{2\pi}{T}$
Wavelength: $\lambda = \nu T = \frac{v}{\nu} = \frac{2\pi v}{\omega}$	
Equation of oscillations: $x = A \sin(\omega t + \varphi_0)$ $x = A \cos(\omega t + \varphi_0)$ A is amplitude; t is time. $\varphi = (\omega t + \varphi_0)$ is phase; φ_0 is initial phase	

Molecular kinetic theory and thermodynamics	
Molar mass: $\mu = \frac{m}{\nu} = m_0 N_A$	Amount of substance (number of moles): $\nu = \frac{m}{\mu} = \frac{N}{N_A}$
Avogadro's number $N_A = 6 \times 10^{23} \text{ mol}^{-1}$ N is the number of particles	Mass of a molecule: $m_0 = \frac{m}{N} = \frac{m}{\nu N_A} = \frac{\mu}{N_A}$
Concentration: $n = \frac{N}{V}$	Density: $\rho = m_0 n = m/V$
Heat capacity: $C = \frac{\Delta Q}{\Delta T} \left[\frac{J}{K} \right]$ Specific heat: $c = \frac{\Delta Q}{m \Delta T} \left[\frac{J}{kg \cdot K} \right]$	Avogadro's law: under normal conditions $(T_0 = 273 \text{ K}, p_0 = 1/01 \times 10^5 \text{ Pa})$ volume of 1 mole of any gas is $V_\mu = 22,4 \times 10^{-3} \frac{m^3}{mol}$
Boltzmann constant: $k = \frac{R}{N_A}$ $k = 1/38 \times 10^{-23} \text{ J/K}$ Gas constant: $R = 8.32 \text{ J/mol} \cdot \text{K}$	Basic MKT equation: $p = \frac{2}{3} n \overline{W}_k$ Average kinetic energy $\overline{W}_k = \frac{m_0 \overline{v}^2}{2}$

Relationship between kinetic energy and temperature: $W_k = \frac{3}{2} kT$	Gas pressure: $p = nkT = \frac{1}{3} nm_0 \bar{v}^2 = \frac{1}{3} \rho \bar{v}^2$ n is concentration.
Gas density: $\rho = \frac{p\mu}{RT}$ μ is molar mass	Root mean square velocity of gas molecules: $\bar{v} = \sqrt{\frac{3kT}{m_0}} = \sqrt{\frac{3RT}{\mu}} = \sqrt{\frac{3p}{\rho}}$
Combined gas law (Clapeyron equation): $\frac{PV}{T} = \text{const}, m = \text{const}$	Ideal gas law (Mendelev —Clapeyron equation): $PV = \frac{m}{\mu} RT$ For 1 mole ($\nu=1$): $\frac{PV\mu}{T} = R$
Isothermal process: $T = \text{const}$ Boyle–Mariotte law: $PV = \text{const}$ ($m = \text{const}$)	
Isobaric process: $P = \text{const}$ Gay-Lussac's law $\frac{V}{T} = \text{const}$ ($m = \text{const}$)	
Isochoric process: $V = \text{const}$ Charles' law: $\frac{P}{T} = \text{const}$ ($m = \text{const}$)	
Dalton's law: the pressure of a gas mixture is equal to the sum of partial pressures, where partial pressure is the pressure a gas would exert in the absence of other gases. $p = p_1 + p_2 + \dots + p_n = \sum_{i=1}^n p_i$	
Internal energy of a gas	$\Delta U = \frac{3}{2} \frac{m}{\mu} R \Delta T$, For 1 mole $U_\mu = \frac{3}{2} RT$
Heating/cooling	$Q = \Delta U = cm\Delta T = cm(T_2 - T_1)$ (J) c is specific heat capacity (J/(kg K))
Melting/solidification (crystallisation) ($T = \text{const}$)	$Q = \lambda m$ λ is the specific heat of melting/solidification (J/kg)
Vaporisation/condensation ($T = \text{const}$)	$Q = rm$ r is the specific heat of vaporisation/condensation (J/kg)

Combustion	$Q = qm$ q is the specific heat of combustion (J/kg)	
Efficiency: T_1 is a heater; T_2 is a refrigerator; Q_1 is received from the heater; Q_2 is given to the refrigerator	$\eta = \frac{A_{used}}{Q_1} = \frac{Q_1 - Q_2}{Q_1} \times 100\%$ $A_{used} = Q_1 - Q_2$ $\eta = \frac{T_1 - T_2}{T_1}$ $\eta < 1 (< 100\%)$	
Law of thermodynamics	$\Delta Q = \Delta U + A$	
First law of thermodynamics	isotherm process	$\Delta T = 0 \rightarrow \Delta U = 0 \rightarrow \Delta Q = A = p\Delta V$
	isobaric process	$\Delta Q = \Delta U + p\Delta V$
	isochoric process	$\Delta V = 0 \rightarrow A = 0 \rightarrow \Delta Q = \Delta U = \frac{3}{2} \frac{m}{\mu} R\Delta T$
	adiabatic process	$\Delta Q = 0 \rightarrow \Delta U = -A$
Gas operation:	$A = p\Delta V = p(V_2 - V_1)$	
Heat balance equation:	$\sum_{i=1}^n (Q_i)_{given\ off} = \sum_{i=1}^n (Q_i)_{received}$ $c_1 m_1 (T_1 - T) = c_2 m_2 (T - T_2)$	
Entropy S	$dS = \frac{dQ}{T}, S = k_B \ln \Omega$ Ω is the total number of microstates	

Conductors			Semiconductors	Dielectrics
conduct electricity well			conduct electric current under certain conditions	do not conduct electric current under any conditions
<i>Metals</i>	<i>Electrolytes</i>	<i>Gases</i>		
$\rho = 10^{-5} - 10^{-8} \Omega \cdot m$			$\rho = 10^4 - 10^5 \Omega \cdot m$	$\rho = 10^{10} - 10^{16} \Omega \cdot m$
Ag, Cu, Ni, Pt, Hg, Fe	Solutions of salts, acids and alkalis	Under ionisation condition	Be, Se, ZnO, Cu ₂ O, Si, Ge, groups IV and V of Mendeleev's periodic table	Quartz, mica, paraffin, porcelain, amber, sulphur, oils, rubber, glass, ebonite, ceramics

Electrodynamics	
Charge $q = Ne$ (C) N is number of particles	Elementary charge (electron) $e = -1.6 \cdot 10^{-19}$ C
Coulomb's law $\vec{F} = \frac{q_1 q_2}{4\pi\epsilon\epsilon_0 r^2} \vec{r}$ $\epsilon_0 = 8.85 \cdot 10^{-12}$ F/m is the dielectric constant of vacuum	Electrical capacity $C = \frac{q}{\varphi}$ (F) Parallel connection: $C = C_1 + C_2$ Serial connection: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$ Energy of a capacitor: $W = \frac{qU}{2} = \frac{CU^2}{2} = \frac{q^2}{2C}$
Field strength: $\vec{E} = \frac{\vec{F}}{q} \quad E = \frac{U}{d}$	
An electric dipole consists of two equal but opposite charges. The electric dipole moment vector p points from the negative charge r to the positive charge $\vec{p} = \vec{r}q$	The torque acting on an electric dipole placed in a uniform electric field E is $\vec{\tau} = \vec{p} \times \vec{E}$
Potential $\varphi = \frac{A}{q}$	Voltage $U = \frac{A}{q}$ (B)
The electric flux through the surface is $\varphi_E = \vec{E}\vec{A} = EA\cos\theta$, θ is the angle between the normal to the surface and the field strength vector	
Gauss's Law. The net flux through any closed surface is proportional to the net charge enclosed Q. $\varphi_E = \frac{Q}{\epsilon_0}$	
EMF $\mathcal{E} = A/q$ (V)	$\mathcal{E} = \sum_{i=1}^N \mathcal{E}_i$
Current strength $I = \frac{\Delta q}{\Delta t}$ (A) in metals $I = envS$	Current density $j = \frac{I}{S}$ in metals $j = env$
Resistance $R = \rho \frac{l}{S}$ (Ω) l is the length of conductor $S = \pi r^2 = \frac{\pi d^2}{4}$ is the cross-sectional area	Specific resistance depends on temperature (for metals) $\rho = \rho_0(1 + \alpha\Delta T)$ ($\Omega \cdot m$)

Ohm's law for a section of a circuit $I = \frac{U}{R}$ Ohm's law for a complete circuit $I = \frac{\varepsilon}{R+r}$ <i>r</i> is internal resistance of the source	
Work of current $A = IUt = I^2Rt = \frac{U^2t}{R} \quad (J)$ <i>t</i> (s) is time	Current power $P = \frac{A}{t} = IU = I^2R = \frac{U^2}{R} \quad (W)$ $1 \text{ Wh} = 3600 \text{ J}$
Joule-Lenz law $Q = I^2Rt$	Energy of the electric field $W = \frac{q\varphi}{2} = \frac{C\varphi^2}{2} = \frac{q^2}{2C}$
1 law of electrolysis $m = kq = kI\Delta t$ 2 law of electrolysis $k = A/(Fn)$ <i>k</i> — electrochemical equivalent; <i>A</i> — atomic mass; <i>n</i> — valence Faraday constant $F = e N_A \quad F = 9.65 \cdot 10^4 \text{ C/mol}$ United Faraday's Law Mass of released substance $m = m_{0i}N_i$ ion mass $m_{0i} = \frac{A}{N_A}$ number of ions reaching the electrode $N_i = \frac{\Delta q}{q_{oi}}$ $\Delta q = I\Delta t \quad q_{oi} = \bar{e}n$	
ionisation condition $\frac{mv^2}{2} \geq A_i$ ionisation energy $W_i = eU_i$ U_i is ionisation potential difference	
Intrinsic conductivity: n-type — electron, p-type — hole. Impurity conductivity: Donor, easily giving up electrons, in n-type semiconductors; Acceptor, increasing the number of holes, in p-type semiconductors	

<i>Series connection</i>		<i>Parallel connection</i>	
Current strength does not change	$I = I_1 = I_2$ $I = \frac{\varepsilon_1 + \varepsilon_2}{R + r_1 + r_2}$ $= \frac{n\varepsilon}{R + nr}$	Current strength is equal to the sum of the currents	$I = I_1 + I_2$ $I = \frac{\varepsilon}{R + \frac{r}{n}}$ $\frac{I_1}{I_2} = \frac{R_2}{R_1}$

Voltage is equal to the sum of voltages	$U = U_1 + U_2$ $\frac{U_1}{R_1} = \frac{U_2}{R_2}$	Voltage does not change	$U = U_1 = U_2$
Overall resistance increases	$R = R_1 + R_2$	Overall resistance decreases	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ $\lambda = \lambda_1 + \lambda_2$

Magnetism	
Ampere strength (per current-carrying conductor)	$F_A = IlB\sin\alpha$
Lorentz force (on a charged particle)	$F_L = qvB\sin\alpha$
Relationship between magnetic induction and magnetic field strength $B = \mu\mu_0 H$ <i>Magnetic constant</i> $\mu_0 = 1.26 \times 10^{-6} \text{ (B}\cdot\text{s)/(A}\cdot\text{m)}$, $\mu_0 = 4\pi \times 10^{-7} \text{ GN/m}$	
Newton's second law for a particle of charge q and mass m , moving with speed v in a direction perpendicular to a magnetic field B is $\frac{mv^2}{r} = qvB$ $T = \frac{2\pi m}{qB}$ is a period of rotation around magnetic field B	
The magnetic field of a current: $B = \frac{\mu_0}{2\pi r} I$	
Solenoid induction: $B = \mu\mu_0 \ni \text{ (T)}$	Magnetic flux $\varphi = BS\cos\alpha \text{ (Wb)}$
EMI Law: $\mathcal{E}_i = \frac{-\Delta\varphi}{\Delta t}$	EMF of self-induction: $\mathcal{E}_{si} = -L \frac{\Delta I}{\Delta t}$ L is inductance (H)
Magnetic field energy: $W = \frac{LI^2}{2}$	Work of magnetic field: $A = I\Delta\varphi$
Alternating current (AC)	
Current strength Amplitude (maximum) value Effective value Instantaneous value	$I = I_0 \sin(\omega t + \varphi_0)$ $I_0 = I_{max} = q_0 \omega$ $I_{\text{eff}} = I_A = \frac{I_0}{\sqrt{2}}$ $i = I_{max} \cos \omega t$

Voltage	$U = U_0 \sin(\omega t + \varphi_0)$
Amplitude (maximum) value	$U_0 = U_{max} = \frac{q_0}{C}$
Effective value	$U_{\text{э}} = U_{\text{д}} = \frac{U_0}{\sqrt{2}}$ $U = U_m \cos \omega t$
Instantaneous value	
Charge	$q = q_0 \sin(\omega t + \varphi_0)$
EMF	$\mathcal{E} = \mathcal{E}_0 \sin(\omega t + \varphi_0)$
Inductive reactance $X_L = \omega L$	Capacitance $X_C = \frac{1}{\omega C}$
Total resistance (impedance) is the sum of active (ohmic) and reactive components: $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$	
Natural frequency of oscillations: $\omega_0 = \frac{1}{\sqrt{LC}}$	
Thomson's formula: $T = 2\pi\sqrt{LC}$ (period)	
Resonance condition: $\omega L = \frac{1}{\omega C}$	
Attenuation coefficient: $\delta = \frac{R}{2L}$	
Decrement of attenuation: $\beta = \frac{1}{\tau}$ (τ is the relaxation time required for amplitude to decrease by a factor of e)	
$\chi = \beta T = \frac{1}{N}$ (N is the number of oscillations during which the amplitude decreases by a factor of e)	
Energy in the circuit: $\frac{CU_m^2}{2} = \frac{LI_m^2}{2}$	
Energy loss: $W = I^2 R t$	
Transformation ratio: $K = \frac{N_1}{N_2} = \frac{E_1}{E_2} = \frac{U_1}{U_2}$	
$K > 1$: step-down transformer	
$K < 1$: step-up transformer	
Efficiency is the ratio of power in the secondary and primary windings: $\eta = \frac{P_2}{P_1} \times 100\%$	

Effects of a current		
Threshold of perceptible current (0.6—1.5 mA)	Lowest current whose effect is felt (itching, tingling). The threshold depends on the current frequency, the contact location and the area of voltage application	Harmful under prolonged exposure
Non-release current threshold (10—15 mA).	Pain and cramps. Affects the exposed area and the entire body	Blood vessels constrict, blood pressure rises, consciousness is lost, and prolonged exposure causes respiratory arrest
Fibrillation (more than 100 mA)	After one to two seconds, contraction of the heart muscle fibres intensifies	Without defibrillation, irreversible damage to brain tissue occurs within 10 minutes
Current above 5A		Immediate cardiac arrest
Up to U values less than 250—300 V		Direct current is four to five times safer than alternating current
At U above 300 V		The danger of direct current increases
Frequencies below 1000 Hz are electrical-ly hazardous, with the risk decreasing as the frequency rises		A stimulus occurring during the cardiac contraction phase can cause resonance and fibrillation
A frequency of approximately 450 kHz		The current passes through the insensitive surface layers of the skin

Optics	
Geometrical optics	
Law of reflection:	$\alpha = \alpha'$
Law of refraction:	$\frac{\sin \alpha}{\sin \beta} = \frac{n_2}{n_1}$
Brewster's angle	$\operatorname{tg} \alpha_B = n_{21}$

Optical power of a lens in a medium: $D = (n - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (n = n_{\text{lens}} / n_{\text{medium}})$ $D = \frac{1}{F}, \text{ focal length: } F = \frac{R}{2(n-1)}$		
Thin lens formula: $\pm \frac{1}{F} = \frac{1}{a} \pm \frac{1}{b}$ Lens magnification factor: $k = \frac{b}{a} = \frac{H}{h}$ <i>a</i> is the distance from the lens to the object. <i>b</i> is the distance from the lens to the image. <i>H</i> is the image size; <i>h</i> is the object size		
For a lens system: $D = D_1 + D_2 + \dots$ $k = k_1 \times k_2 \times \dots$		
Relationship between the radius of curvature and the focal length of a spherical mirror: $F = R/2$		
Microscope magnification factor: $k = \frac{L\Delta}{F_1 F_2}$ $L \approx 0.25$ m is the distance of best vision. ΔL is the distance between the focal planes of the objective and the eyepiece. Microscope resolution limit: $Z = \frac{0.5\lambda}{\sin \frac{\phi}{2}} = \frac{0.5\lambda_0}{n \sin \frac{\phi}{2}}$ where $\lambda_0 = n \lambda$ is the wavelength of light in a vacuum. The angular aperture is the angle between the outer rays of the light beam entering a lens. Numerical aperture: $A = n \sin \frac{\phi}{2}$		
Converging lens: $D > 0, F > 0$	$F = 1$ m	$D = 1$ dioptre
	$F > 1$ m	$D < 1$ dioptre
	$F < 1$ m	$D > 1$ dioptre
Diverging lens: $D < 0, F < 0$	The focus is virtual. The image is always reduced, virtual and direct	

Photometry		
Value	Formulas	Units of measurement
		Light units Energy units

For an illuminated surface of the site:			
Luminous flux	$\Delta\varphi = \frac{\Delta W}{\Delta t}$	Lm Lumen	W
Surface illumination	$E = \frac{\varphi}{S}$	Lx lux	W/m ²
Power of light	$I = \frac{\varphi}{\Omega} \cos\alpha$	Candela cd	W/steradian
Spatial angle	$\Omega = \frac{S}{r^2}$		steradian
For a luminous surface of the source			
Brightness	$B = \frac{I}{S \cos\alpha}$	Cd/m ²	W/(steradian·m ²)
Radiant emittance	$R = \frac{\varphi}{S}$	Lm/m ²	W/m ²
Emissivity is spectral radiance	$dR_v = r_v \, dv \, W \, s / m^2$		

Electromagnetic waves

Absolute refractive index of a medium: $n = \frac{c}{v}$

Relative refractive index of a medium: $n_{21} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$

Speed of light in vacuum: $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ $c = 3 \cdot 10^8$ m/s

Speed of light in matter: $v = \frac{c}{\sqrt{\varepsilon\mu}}$ $v = \frac{\lambda}{T} = \lambda\nu = \lambda \frac{\omega}{2\pi}$

Wavelength: $\lambda = vT = \frac{v}{\nu}$ Frequency: $\omega = 2\pi\nu = 2\pi/T$

Energy flux density: $\bar{S} = [\vec{E} \times \vec{H}]$

magnitude of energy flux density for a plane wave:

$$S \nu = \sqrt{\frac{\varepsilon_0}{\mu_0}} E^2 = c \varepsilon_0 E^2 = \frac{1}{120\pi} E^2 \quad (\text{W/m}^2)$$

Decrement of attenuation: $\delta = \ln \frac{x(t)}{x(t+T)} = \beta T$

β is the damping coefficient related to phase velocity:

$$v_\phi = \frac{\omega}{\beta} = \frac{1}{\sqrt{L_0 C_0}}$$

Total decay time: $t = T/\delta$, period $T = \lambda / \nu$

Penetration depth: $\delta = \sqrt{\frac{1}{\pi\mu\mu_0\lambda\nu}}$

$\lambda = 1/\rho$ is conductivity $(\Omega m)^{-1}$, ρ is resistivity (Ωm)

Interference

Condition for maximum interference: the path difference equals the integer number of wavelengths (or an even number of half-waves).

$$\delta = \Delta l = 2k \frac{\lambda}{2} = k\lambda \quad (k = 0, 1, 2, \dots)$$

$$\Delta\varphi = 2k\pi \quad \cos\Delta\varphi = 1$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \quad I_{\max} > (I_1 + I_2)$$

Condition for minimum interference: the path difference equals the odd number of half-waves.

$$\delta = \Delta l = (2k + 1) \frac{\lambda}{2} \quad (k = 0, 1, 2, \dots)$$

$$\Delta\varphi = (2k + 1)\pi \quad \cos\Delta\varphi = -1$$

$$I = I_1 + I_2 - 2\sqrt{I_1 I_2} \quad I_{\min} < (I_1 + I_2)$$

Distance between maxima: $x_m = \frac{m\lambda a}{b}$

Diffraction

Maximum condition for a diffraction grating

$$d \sin \varphi = k \lambda$$

$k = \pm 0, 1, 2 \dots$ is the order of the spectrum (principal maxima).

Resolving power of a diffraction grating: $R = kN = \frac{\lambda}{\Delta\lambda}$

N is the number of lattice slots.

$\Delta\lambda$ is the minimum wavelength difference.

Polarisation

Malus's law $I = I_0 \cos^2 \varphi$, φ is the angle between polarisers

Laws of thermal radiation

Stefan — Boltzmann law for a black body

$$R(T) = \sigma T^4$$

$\sigma = 5.7 \times 10^{-8} \text{ (W} \cdot \text{m}^{-2} \cdot \text{K}^{-4})$ is the Stefan — Boltzmann constant.

For a grey body, $R(T) = \alpha \sigma T^4$ (α is the absorption coefficient).

Wien's displacement law: as temperature increases, the maximum emissivity shifts towards shorter wavelengths.

$$T \lambda_{\max} = b \text{ or } T c / \nu_{\max} = b; \quad b = 2.9 \times 10^{-3} \text{ (m} \cdot \text{K)} \text{ is Wien's constant.}$$

Rayleigh-Jeans law: $r_{\nu,T} = \frac{2\pi\nu^2}{c^3} kT$

Planck's formula: $r_{\nu,T} = \frac{2\pi\nu^2}{c^3} \frac{h\nu}{\exp(\frac{h\nu}{kT}) - 1}$

Electromagnetic wave scale			
Name	Wave length	Frequency	Source
Extra long	More 10 km	Less 30 kHz	Emitted due to the collective oscillatory motion of electrons in technical devices such as antennas or in natural formations like lightning, solar currents, and currents in the ionosphere. The size of the antenna system is approximately equal to λ .
Long waves	10 km—1 km	3×10^4 — 3×10^5 Hz	
Average	1000 m—100 m	3×10^5 — 3×10^6 Hz	
Short waves	100 m—10 m	3×10^6 — 3×10^7 Hz	
Ultra short waves	10 m—1 m	More 30 mHz	
ultra short waves	10^{-2} — 10^{-4} m	More 300 mHz	
Infra red	10^{-4} — 10^{-5} m	3×10^{11} — 3×10^{14} Hz	Emitted due to the thermal motion of crystallisation centres (λ is approximately equal to the distance between ions)
Visible range	10^{-6} — 10^{-7} m 380 nm—780 nm	750×10^{12} — 380×10^{12} Hz	Thermal radiation from the sun and other sources, fluorescence, chemical reactions and LEDs occur due to the transition of electrons from the first shell under external influences and their return, emitting a quantum of light
Ultraviolet	10^{-8} — 10^{-9} m	8×10^{14} — 3×10^{16} Hz	
X-ray	soft X-ray	3×10^{16} — 6×10^{19} Hz	Emitted due to electrons in the inner shells of atoms and the deceleration of fast-moving particles
	Hard X-ray		
Gamma radiation	$< 10^{-2}$ nm	$> 3 \cdot 10^{18}$ Hz	Emitted as a result of nuclear reactions, with the nucleus size being less than 10^{-12} m

Quantum physics
For any chemical element A_ZX , $A=Z+N$ Z is the atomic number, equal to the number of protons or electrons. A is the mass number of the element, and N is the number of neutrons
Elementary particles: ${}_{-1}^0e$, ${}_{+1}^0e$, 1_1p , 1_0n , ${}^4_2\alpha$
Photon energy: $E = h\nu = \frac{hc}{\lambda}$ $h = 6.62 \times 10^{-34} \text{ J}\cdot\text{s}$ is Planck's constant
Binding energy of the atomic nucleus: $\Delta E = \Delta mc^2$ Mass defect: $\Delta m = Zm_p + (A-Z)m_n - m_{\text{total}}$
Rutherford atomic model: $m_e a_c = m_e \frac{v^2}{R} = \frac{e^2}{4\pi\epsilon_0 R^2}$
Energy in the n^{th} orbit $W = \frac{-m_e e^4}{8n^2 \epsilon_0^2 h^2}$ $W = \frac{-13.6}{n^2} \text{ (eV)}$ $W \frac{1}{n^2}$
Radiation frequency: $\nu = R \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$ $R = 3.29 \times 10^{15} \text{ Hz}$ is the Rydberg constant
Einstein's equation for the photoelectric effect: $h\nu = A + \frac{mV^2}{2}$, where A is the work function.
Red border of the photoelectric effect: $\lambda_{red} = \frac{hc}{A}$ $\nu_{red} = \frac{A}{h}$
Light pressure: $P = \frac{I}{c} (1 + \rho)$; I is light intensity; reflection coefficient: $\rho_{\text{white}} = 1$, $\rho_{\text{black}} = 0$.
Shift rule for α -decay (two positions to the left, towards the beginning of the periodic table): ${}^A_ZX \rightarrow {}^4_2\text{He} + {}^{A-4}_{Z-2}Y$
Shift rule for β -decay (one position to the right): ${}^A_ZX \rightarrow {}^A_{-1}e + {}^{A}_{Z+1}Y$
Law of radioactive decay: $N = N_0 e^{-\lambda T}$ or $N = N_0 2^{-\frac{T}{T}}$ N_0 is the initial number of nuclei; $\Delta N = -\lambda N \Delta t$ is the number of decayed nuclei. The half-life T is the time interval during which the number of radioactive nuclei decreases to half of its original value. ($N = N_0/2$). $T = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} = 0.693\tau$

Mean life time τ is the time during which the number of undecayed nuclei decreases by a factor of $e \approx 2.72$.

$$\tau = \frac{1}{\lambda} = \frac{T}{0.69} = 1.44T$$

λ (s^{-1}) is decay constant

Activity of a radioactive isotope (decay rate):

$$A = \frac{-\Delta N}{\Delta t} = \lambda N = \lambda N_0 e^{-\lambda t}$$

Becquerel (Bq) = s^{-1} ,

Curie (Ci) = 3.7×10^{10} Bq

At $t = T$, the activity is higher the greater the number of radioactive nuclei

N and the shorter the half-life T : $A = \frac{N}{T} \ln 2$

Solid density		
	<i>kg/m³</i>	<i>g/cm³</i>
Osmium	22 600	22.6
Iridium	22 400	22.4
Platinum	21 500	21.5
Gold	19 300	19.3
Lead	11 300	11.3
Silver	10 500	10.5
Copper	8900	8.9
Brass	8500	8.5
Steel, iron	7800	7.8
Tin	7300	7.3
Zinc	7100	7.1
Cast iron	7000	7.0
Corundum	4000	4.0
Aluminium	2700	2.7
Marble	2700	2.7
Window glass	2500	2.5
Porcelain	2300	2.3
Concrete	2300	2.3
Brick	1800	1.8
Refined sugar	1600	1.6
Plexiglas	1200	1.2
Nylon	1100	1.1

	kg/m^3	g/cm^3
Polyethylene	920	0.92
Paraffin	900	0.90
Ice	900	0.90
Oak (dry)	700	0.70
Pine (dry)	400	0.40
Cork	240	0.24

Dosimetry			
Absorbed dose	$D=W/m$	J/kg=Gy	The absorbed dose is the ratio of radiation energy to the mass of the irradiated substance. The unit of absorbed dose is the gray (Gy)
Equivalent dose	$H=k\cdot D$	Sv (sievert)	Assesses the biological effects of radiation: 1 Sv=1 J/kg
Quality factor	k		Indicates how much greater the hazard is compared to γ -radiation.
Exposure dose	P	C/kg	The ratio of the total charge of ions to the mass of air in a volume; determines the degree of ionisation of air when exposed to Rg or γ -rays
Dose power	$N=D/t$	R (roentgen)	Dose per unit of time: 1R= 2.58×10^{-4} C/kg One roentgen is the amount of gamma or X-radiation required to produce approximately 2.08×10^9 ion pairs in 1 cm ³ of dry air at standard temperature and pressure. 00 rem = 1 sievert
		Gy /s	Absorbed dose rate: 1Gy=100R
		A/kg	The exposure dose rate depends on the number of photons emitted from a source in all directions. The number of photons is directly proportional to the activity and inversely proportional to the surface area
Biological dose		Bq 10^{-2} J/kg	Absorbed radiation energy equivalent to 1R

Resistivity and Conductivity at 20 °C (*)

Material	ρ ($\Omega \cdot m$) at 20 °C Resistivity	σ (S/m) at 20 °C Conductivity
Silver	1.59×10^{-8}	6.30×10^7
Copper	1.68×10^{-8}	5.96×10^7
Annealed copper	1.72×10^{-8}	5.80×10^7
Gold	2.44×10^{-8}	4.10×10^7
Aluminium	2.82×10^{-8}	3.5×10^7
Calcium	3.36×10^{-8}	2.98×10^7
Tungsten	5.60×10^{-8}	1.79×10^7
Zinc	5.90×10^{-8}	1.69×10^7
Nickel	6.99×10^{-8}	1.43×10^7
Lithium	9.28×10^{-8}	1.08×10^7
Iron	1.0×10^{-7}	1.00×10^7
Platinum	1.06×10^{-7}	9.43×10^6
Tin	1.09×10^{-7}	9.17×10^6
Carbon steel	(10^{10})	1.43×10^{-7}
Lead	2.2×10^{-7}	4.55×10^6
Titanium	4.20×10^{-7}	2.38×10^6
Grain oriented electrical steel	4.60×10^{-7}	2.17×10^6
Manganin	4.82×10^{-7}	2.07×10^6
Constantan	4.9×10^{-7}	2.04×10^6
Stainless steel	6.9×10^{-7}	1.45×10^6
Mercury	9.8×10^{-7}	1.02×10^6
Nichrome	1.10×10^{-6}	9.09×10^5
GaAs	5×10^{-7} to 10×10^{-3}	5×10^{-8} to 10^3
Carbon (amorphous)	5×10^{-4} to 8×10^{-4}	1.25 to 2×10^3
Carbon (graphite)	2.5×10^{-6} to 5.0×10^{-6} // basal plane $3.0 \times 10^{-3} \perp$ basal plane	2 to 3×10^5 //basal plane $3.3 \times 10^2 \perp$ basal plane
Carbon (diamond)	1×10^{12}	$\sim 10^{-13}$
Germanium	4.6×10^{-1}	2.17
Sea water	2×10^{-1}	4.8
Drinking water	2×10^1 to 2×10^3	5×10^{-4} to 5×10^{-2}

End of the Table

Material	ρ ($\Omega \cdot m$) at 20°C Resistivity	σ (S/m) at 20°C Conductivity
Silicon	6.40×10^2	1.56×10^{-3}
Wood (damp)	1×10^3 to 4	10^{-4} to 10^{-3}
Deionised water	1.8×10^5	5.5×10^{-6}
Glass	10×10^{10} to 10×10^{14}	10^{-11} to 10^{-15}
Hard rubber	1×10^{13}	10^{-14}
Wood (oven dry)	1×10^{14} to 16	10^{-16} to 10^{-14}
Sulphur	1×10^{15}	10^{-16}
Air	1.3×10^{16} to 3.3×10^{16}	3×10^{-15} to 8×10^{-15}
Paraffin wax	1×10^{17}	10^{-18}
Fused quartz	7.5×10^{17}	1.3×10^{-18}
Teflon	10×10^{22} to 10×10^{24}	10^{-25} to 10^{-23}

* <https://sciencenotes.org/table-of-electrical-resistivity-and-conductivity/>

IV. TEST QUESTIONS, TASKS AND EXERCISES

Preparation tasks

Topic 1. Kinematics

1. What is a material point?
2. What is a frame of reference? Does the description of motion depend on the choice of the reporting system?
3. Explain the difference between path and displacement.
4. What is the difference between speed and velocity?
5. What is acceleration?
6. Explain the difference between uniform and accelerated motion.
7. Explain the difference between rectilinear and curvilinear motion.
8. Explain tangential and normal acceleration in curvilinear motion.
9. What are the rules for adding vector quantities?
10. What is a period, and how is it related to frequency?
11. What is the graph of displacement in uniform and accelerated motion?
12. What is the graph of velocity in uniform, accelerated, and decelerated motion?

Topic 2. Dynamics

1. What is the difference between inertia and inertia?
2. What is the difference between mass and weight? What is the difference between weight and gravity?
3. Explain how acceleration due to gravity varies with height above the Earth.
4. Explain external and internal friction, the factors on which friction force depends, and the types of mechanical friction.

5. Explain deformation, the nature of elastic force and Hooke's law.

6. Explain the factors that determine the value of work, the conditions under which work is zero and the relationship between work and energy.

7. Explain the types of energy, the law of conservation of mechanical energy, and the reasons why conservation laws are not satisfied in open systems.

8. What is the difference between elastic and inelastic collisions, and how are the laws of conservation of momentum and energy expressed in each case?

9. Formulate Newton's laws.

10. What is an absolutely rigid body? How is the translational and rotational motion of an absolutely rigid body determined?

11. What is the moment of inertia of a body? What is an analogue in translational motion?

12. What is angular momentum?

13. Explain the basic law of rotational motion dynamics.

14. What is energy and work in rotational motion?

15. Draw the forces acting on a body in the following cases: when it moves up an inclined plane; when it slides freely down an inclined plane; when a car is at the top of a convex bridge; and when a ball oscillates at the lowest point of its trajectory.

Topic 3. Statics. Mechanical properties of bodies

1. Moment of force. Shoulder.

2. Lever. Rule of lever equilibrium.

3. Movable and fixed blocks.

4. Types of equilibrium. Conditions of equilibrium.

5. Thermal expansion of bodies.

6. Types of deformation.

7. Mechanical stress. Young's modulus.

Topic 4. Hydrostatics and hydrodynamics

1. Mechanical pressure. Hydrostatic pressure. Atmospheric pressure.

2. Pascal's law. Hydrostatic paradox.
3. Archimedes' force. Weight of a body in a liquid. Condition of floating bodies.
4. Surface tension.
5. Capillary phenomena. Laplace pressure.
6. Frenkel's theory. Molecular action radius.
7. Ideal fluid. Compressibility.
8. Viscosity: dynamic and kinematic.
9. Dependence of liquid and gas viscosity on temperature.
10. Jet continuity theorem.
11. Bernoulli equation.
12. Jet flow velocity. Torricelli's formula.
13. Volumetric flow velocity. Poiseuille's equation.
14. Hydraulic resistance.
15. Types of flow.
16. Reynolds criterion.
17. Internal friction force. Newton's equation. Velocity gradient.
18. Stokes' law.
19. Free path, effective cross-section and effective diameter.
20. Gas viscosity. Sutherland's formula.

Topic 5. Haemodynamics and rheology of biological fluids

1. Flow characteristics: pressure, volume velocity and hydraulic resistance.
2. Real liquids. Newtonian and non-Newtonian fluids.
3. Types of blood flow in health and disease. Rheological properties of blood.
4. Change in velocity profile depending on vessel radius.
5. Viscosity of biological fluid as a diagnostic factor. Dependence of blood viscosity on vessel radius.
6. Elasticity of erythrocytes. Deformation of erythrocytes in capillaries.
7. Mechanical properties of blood vessels. Lamé's equation.
8. Frank — Starling circulation model.

9. Pulse wave.
10. Measuring blood pressure.

Topic 6. Mechanical oscillations and waves

1. Oscillatory motion.
2. Types of mechanical oscillations.
3. Mathematical and spring pendulums.
4. Causes of the attenuation of free oscillations.
5. Equation and graph of harmonic oscillations.
6. Amplitude, period, frequency and phase of oscillations.
7. Classification of mechanical waves.
8. Elastic wave.
9. Wave front. Huygens — Fresnel principle.
10. Longitudinal and transverse waves.
11. Wave equation. Wave characteristics.
12. Propagation speed and wavelength.
13. Wave resistance of the medium. Acoustic impedance.
14. Energy characteristics.
15. Analysis of wave processes in the human body or biological tissues using medical and biological equipment.

Topic 7. Acoustics

1. Standing waves. Fundamental mode. Overtones.
2. Types of sound.
3. Acoustic spectrum of a complex tone.
4. Sound ranges. Effects of ultrasound and infrasound on the body.
5. Physical characteristics of sound.
6. Characteristics of auditory sensations.
7. Weber — Fechner law.
8. Biophysics of hearing.
9. Sound research methods in medicine.
10. Doppler effect.
11. Shock wave in biophysics.

Topic 8. Ultrasound

1. Physical properties of ultrasound: wavelength, intensity, pressure and vibrational velocity.
2. Propagation of ultrasound in a medium: wave impedance, ultrasound propagation velocity, reflection coefficient, absorption and penetration depth, absorption coefficient.
3. Diffraction and interference of ultrasound waves.
4. Sources and receivers of ultrasound (natural and artificial).
5. Piezoelectric or magnetostrictive sensors.
6. Types of scanning.
7. Interaction of ultrasound with matter: mechanical, thermal and chemical.
8. Diagnostic ultrasound methods. Measuring blood flow velocity. Duplex scanning.
9. Other applications of ultrasound.
10. Advantages and disadvantages of ultrasound methods.

Topic 9. Molecular physics

1. Basic provisions of MKT.
2. Amount of matter. Avogadro's number. Avogadro's law.
3. Temperature. Kelvin scale. Absolute zero.
4. Ideal gas. Ideal gas pressure.
5. Basic equation of MKT.
6. Mendeleev — Clapeyron equation.
7. Gas laws for isoprocesses. Combined gas law.
8. Dalton's law.
9. Internal energy of gas.
10. Amount of heat. Specific heat capacity.
11. Heat capacity of gases at constant volume and at constant pressure. Gas constant.
12. Adiabatic process.
13. Polytropic process.
14. Real gases. Van der Waals equation.
15. Barometric formula.

Topic 10. Thermodynamics

1. States of a TD system.
2. Methods for changing the internal energy of a body.
3. Heat balance equation.
4. Gas work.
5. The first law of thermodynamics and its corollary.
6. First law of thermodynamics in various processes.
7. Phase transitions of the first and second kind.
8. Classification and properties of solids. Dulong — Petit law.
9. Melting and crystallisation. Clausius — Clapeyron equation.
10. Evaporation. Vaporisation and condensation. Boiling.
11. Saturated steam. Humidity. Dew point.
12. Triple point.
13. Combustion. Law of conservation of energy in thermodynamics processes.
14. Second law of thermodynamics. Entropy.
15. The body as a thermodynamic system. Heat balance of the body.

Topic 11. Electrostatics

1. Electrification of bodies. Law of conservation of charge.
2. Interaction of charged particles. Coulomb's law.
3. Electrostatic field. Tension. Principle of superposition of fields.
4. Graphic representation of field distribution in various conductors.
5. Potential. Potential difference. Equipotential surfaces.
6. Relationship between potential difference and voltage.
7. Tension and potential of a charged sphere.
8. Electrical capacitance.
9. Capacitor. Capacitance with various connections of capacitors.
10. Energy of the electric field. Energy of a capacitor.

Topic 12. Direct current

1. Classification of substances by the property of electrical conductivity.
2. Definition of direct current. Direction of current.
3. Action of current.
4. Conditions for the occurrence of electric current.
5. EMF. Voltage. Voltage measurement.
6. Current. Current measurement. Current density.
7. Conductivity in metals. Current in metals.
8. Electrical resistance.
9. Specific resistance and its dependence on the conductor temperature. Temperature coefficient of resistance.
10. Conductivity of various biological tissues. Specific conductivity. Superconductivity.
11. Electric circuits and conductor connections.
12. Ohm's law for a section of a circuit and for a complete circuit. Graphs showing the dependence of current on voltage.
13. Current work. Joule — Lenz law.
14. Current power.
15. Current in liquids. Dissociation and recombination. Dynamic equilibrium. Degree of dissociation.
16. Ionic conductivity. Anions and cations. Temperature coefficient of resistance in liquids.
17. Electrolysis. Faraday's laws of electrolysis. Application of electrolysis.
18. Current in gases. Current in vacuum.
19. Semiconductors. Intrinsic and impurity (donor and acceptor) conductivity. p-n junction.
20. Dielectrics. Polarisation of dielectrics.

Topic 13. Magnetism

1. Magnetic field. Detection of magnetic field.
2. Permanent magnets.
3. Earth's magnetic field.

4. Magnetic induction and magnetic field strength.
5. Direction of magnetic induction.
6. Magnetic permeability of vacuum and matter. Ampère's hypothesis.
7. Diamagnets, paramagnets and ferromagnets.
8. Graphic representation of the magnetic field for various conductor configurations.
9. Ampère's law. Direction of the Ampère's force.
10. Interaction of currents.
11. Lorentz force. Direction of the Lorentz force.
12. Motion of a charged particle in a magnetic field.
13. Phenomenon of electromagnetic induction.
14. Magnetic flux.
15. Induction current.
16. Lenz's rule.
17. Law of EMI.
18. Self-induction.
19. Inductance.
20. Energy of magnetic field.

Topic 14. Electromagnetic oscillations

1. Physical nature of EM oscillations. Oscillatory circuit.
2. Equation and graph of EM oscillations.
3. Characteristics of EM oscillations.
4. Oscillations of charge and voltage on capacitor plates.
5. Oscillations of current. Instantaneous value of current.
6. Effective and amplitude values of current and voltage.
7. Simplest induction generator.
8. Alternating current in a circuit containing a capacitor. Capacitive resistance.
9. Inductor in an AC circuit. Inductive reactance.
10. Total resistance in an AC circuit.
11. Thomson's formula.
12. Ohm's law for an AC circuit.
13. Resonance in an electric circuit.

14. Energy losses.
15. Transformer. Transformation ratio. Transformer efficiency.

Topic 15. Rheography of organs and tissues

1. Conduction currents and displacement currents.
2. Perceptible, non-releasing and fibrillation currents.
3. Dependence of current action on frequency.
4. Rheographic diagnostic methods.
5. Tissue resistance at direct current.
6. Tissue capacitance properties.
7. Tissue polarisation coefficient.
8. Evaluation of the state of biological tissue.
9. Therapeutic methods used in cases of electrical exposure.
10. Effect of magnetic fields on the body.
11. Therapeutic methods based on the action of magnetic fields.
12. Diagnostic methods based on the action of magnetic fields.

Topic 16. Electromagnetic waves

1. Properties and characteristics of EMF. EMF scale.
2. Physical nature of light. White light spectrum. Dispersion.
3. Photometric quantities. Two scales of units in photometry.
4. Interference. Conditions of occurrence. Coherence. Simplest interference schemes.
5. Interference in thin films. Application of interference in medicine.
6. Diffraction. Types of diffraction. Conditions of occurrence.
7. Diffraction grating. Rayleigh criterion. Application of diffraction in medical and biological research.
8. Polarisation. Types of polarisation. Methods for obtaining polarised light. Malus's law.
9. Double refraction. Anisotropy. Optically active substances. Polarimetry.
10. Application of polarisation. Photoelasticity. Saccharimetry.
11. Characteristics of thermal radiation.
12. Laws of thermal radiation.

13. Use of infrared radiation. Thermography.
14. Effect of electromagnetic waves on biological tissue. Medical methods based on the use of various ranges of electromagnetic waves.
15. The body's physical fields.

Topic 17. Geometrical optics

1. Laws of geometrical optics. Limits of applicability of geometrical optics.
2. Refractive index of a medium.
3. Total internal reflection. Brewster's angle. Application of fibre optics in medical and biological research.
4. Lenses. Types of lenses.
5. Main and secondary optical axes, main and secondary foci, and focal planes of the lens.
6. Focal length and optical power of collecting and diverging lenses. Dependence of the focal length of the lens on the refractive index of a medium.
7. Thin lens formula.
8. Methods for determining the magnification factor of a lens.
9. Optical power and magnification factor of a lens system.
10. Methods for constructing images in converging and diverging lenses.
11. Image characteristics.
12. Lens aberrations.

Topic 18. Optical system of the eye

1. Light-conducting and light-receiving apparatus of the human eye.
2. Functions of the cornea, pupil, lens, vitreous body and retina.
3. Accommodation.
4. Colour perception. Visual pathologies associated with rod and cone deficiency.
5. Visual aberrations.
6. Correction of myopia and hyperopia.

7. Optical instruments. Magnification. Resolution limit. Resolving power. Angular and numerical aperture.

8. Glasses have an optical power of -2 dioptres and $+3.5$ dioptres. Which vision defect is corrected by these glasses, what type of lenses do they contain (converging or diverging), and what are their focal lengths?

9. Features of vision of various animals.

10. Prevention of eye diseases.

Topic 19. Basic concepts of quantum physics

1. Energy, momentum and mass of a photon.

2. Planetary model of an atom.

3. Bohr's postulates.

4. Spectrum of a hydrogen atom.

5. Types of spectra. Spectral analysis.

6. Photoelectric effect. Einstein's equation for the photoelectric effect. Red boundary of the photoelectric effect.

7. Light pressure.

8. Compton effect.

9. De Broglie hypothesis. Wave properties of particles.

10. Electron microscope.

11. Stimulated emission. Laser.

12. Application of laser radiation in medicine.

13. X-ray radiation. Types. Properties.

14. Interaction of X-ray radiation with matter.

15. X-ray diagnostics.

Topic 20. Elements of nuclear physics. Dosimetry

1. Nuclear composition. Elementary particles.

2. Nuclear binding energy. Nuclear mass defect.

3. Radioactivity. RA radiation. Types. Properties.

4. Nuclear reactions. Law of conservation of charge number and mass number.

5. Shift rule for α -decay and β -decay.

6. RA decay law.

7. Half-life. Isotope lifetime.
8. Chain reaction.
9. RA isotope activity.
10. Dosimetry. Doses. Units of measurement.
11. Methods of protection from RA radiation.
12. Natural RA background.
13. Natural and anthropogenic factors of RA pollution of the biosphere. Consequences of RA impact on the biosphere.
14. Use of radionuclides.
15. Therapeutic and diagnostic methods using radionuclides.

V. ORAL EXAMINATION QUESTIONS FOR GENERAL MEDICINE STUDENTS

Part 1

Dynamics. Acoustics. Ultrasound

1. Newton's laws.
2. Langevin equation.
3. Diffusion phenomenon. Fick's law.
4. Dynamic and kinematic viscosity. Blood viscosity as a diagnostic factor.
5. Newtonian and non-Newtonian fluids. Blood is a non-Newtonian fluid.
6. Hydrodynamic characteristics of blood flow through vessels: pressure, velocity and hydraulic resistance.
7. Volume velocity. Poiseuille's equation.
8. Types of flows. Reynolds number.
9. Causes of alterations in blood flow.
10. Mechanical properties of vessels. Lamé's equation.
11. Capillary phenomena.
12. Classification of mechanical waves.
13. Standing waves. Fundamental mode and overtones.
14. Wave equation. Wave characteristics.
15. Energy characteristics of waves.
16. Acoustic wave and their spectrum.
17. Physical characteristics of sound. Types of sound.
18. Characteristics of auditory sensations. Weber — Fechner law.
19. Acoustic impedance.
20. Sound research methods in medicine.
21. Reflection, refraction and absorption of ultrasound.
22. Wave impedance. Penetration coefficient.
23. Doppler effect. Dopplerography. Doppler frequency shift.

24. Echo methods.
25. Scope, advantages and disadvantages of ultrasound diagnostics.
26. Therapeutic use of ultrasound.
27. Ranges, intensity and modes of ultrasound exposure.
28. Factors of ultrasound exposure. Contraindications for the use of ultrasound exposure.

Part 2

Effects of electric and magnetic radiation on the body.

Optical range

1. Electrography: its definition, types and objectives.
2. Electric and current dipole.
3. Einthoven's theory.
4. Effects of electric current on the human body. Threshold and maximum permissible current parameters.
5. Impedance of biological tissues.
6. Rheography of organs and tissues. Impedance dispersion.
7. Therapeutic methods involving the use of electric current.
8. Characteristics of the magnetic field.
9. Diagnostic methods based on the effects of magnetic fields.
10. Therapeutic methods based on the action of magnetic fields.
11. Definition, characteristics and properties of electromagnetic waves.
12. Ranges of the electromagnetic wave scale.
13. Interference phenomenon. Condition for observing maximum and minimum intensity.
14. Diffraction. Diffraction in a solution.
15. Polarisation. Types of polarisation. Malus's law.
16. Physical processes in tissues under the influence of electromagnetic waves of different frequency ranges.
17. Diagnostic methods using radio waves.
18. Diagnostic methods using optical radiation.

19. Diagnostic methods using gamma radiation.
20. Physical fields of the body.
21. Methods based on interference.
22. Methods based on diffraction.
23. Methods based on polarisation.
24. Basic photometric quantities.
25. Two scales for measuring photometric quantities.
26. Laws of geometric optics.
27. Types and characteristics of lenses. Characteristics of images in lenses.
28. Optical system of the eye. Visual aberrations and methods of their correction.

Part 3
Thermal radiation. X-ray radiation.
Radioactivity and dosimetry

1. Thermal radiation and its characteristics.
2. Experimental laws describing the dependence of the spectral density of thermal radiation on wavelength.
3. Thermoregulation and its types.
4. Absolutely black, white and grey bodies.
5. Kirchhoff's law.
6. Stefan — Boltzmann law.
7. Wien's displacement law.
8. Classical model explaining thermal radiation patterns. Rayleigh-Jeans law.
9. Quantum-mechanical model explaining thermal radiation patterns. Planck's formula.
10. Radiative loss by humans.
11. Thermography method.
12. Ionising radiation.
13. Bremsstrahlung: spectrum and mechanism of occurrence.
14. Differences in the properties of hard and soft X-ray radiation.

15. X-ray radiation: spectrum and mechanism of generation.
16. Moseley's law.
17. Interaction of electromagnetic radiation with matter. Coherent radiation. Einstein's equation for a two-level system.
18. Effects proving the quantum nature of light. Compton effect and photoelectric effect. De Broglie wavelength.
19. Passage of radiation through matter. Lambert — Bouguer law.
20. X-ray diagnostics.
21. Natural and artificial radioactivity; hazards to the biosphere.
22. Binding energy and nuclear mass defect.
23. Nuclear reaction equation. Examples. Conservation laws in nuclear reactions.
24. Types of radioactive decay. Rule of displacement of nuclei in radioactive decay.
25. Half-life and law of radioactive decay.
26. Absorbed dose, equivalent dose and exposure; corresponding units of measurement.
27. Safe and hazardous dose rate; methods of protection.
28. Activity of a radioactive preparation.
29. Penetrating power of ionising radiation.
30. Methods of radioisotope therapy and radioisotope diagnostics. Labelled atom method.

VI. PROBLEMS

Task 1

Transfer the treatment and diagnostic methods from the list into Table 1, placing them in the corresponding cells.

№	Section of physics	Diagnostic methods	Treatment methods
1111	Haemodynamics		
2222	Acoustics		
3333	Ultrasound		
4444	Electrography of organs and tissues		
5555	Effect of currents on the body		
6666	Effect of electric fields		
7777	Effect of magnetic fields		
8888	Effect of electromagnetic waves		
9999	Interference		
10110	Diffraction		
11111	Polarisation		
12112	Optical range		
13113	Thermal radiation		
14114	X-ray radiation		
15115	Radioactive radiation		

1. Audiometry.
2. Auscultation.
3. Percussion.
4. Phonography.

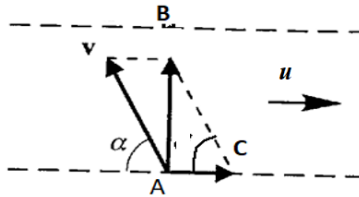
5. Weber — Fechner Law.
6. Acoustic impedance.
7. Evaluation of blood flow parameters by Re.
8. Lamé's equation.
9. Dopplerography.
10. Dopplerometry.
11. Echoencephalography.
12. Echocardiography.
13. Doppler echocardiography.
14. Ultrasound flowmetry (blood flow velocity assessment).
15. Ultrasound ultrasonography (soft tissues).
16. Ultrasound location method (ophthalmology).
17. Ultrasound scanning method (obstetrics).
18. High-frequency ultrasound therapy.
19. High-intensity ultrasound therapy (osteosynthesis).
20. Low-intensity ultrasound therapy.
21. Rheoencephalography.
22. Rheocardiography.
23. Impedance dispersion.
24. ECG.
25. EEG.
26. EMG.
27. Magnetocardiography.
28. Magnetoencephalography.
29. UHF therapy.
30. Darsonvalisation.
31. Diathermy.
32. Diathermocoagulation.
33. Diathermotomy.
34. Electrophoresis.
35. Phonophoresis.
36. Inductothermy.
37. Thermography.
38. Laser treatment.

39. Optical clearing.
40. Interference microscopy.
41. X-ray structural analysis.
42. Gastroscopy.
43. Photoelasticity assessment.
44. Polarimetry.
45. Fluoroscopy.
46. Radiography.
47. Radioisotope teletherapy.
48. Brachytherapy.
49. Labelled atom method.
50. X-ray tomography.

Test. Questions and Problems

Topic 1. Mechanics. Hydrostatics and hydrodynamics

1. What is the nature of a body's motion if it is described by the equation $S(t) = 15 + 5t + 4t^2$? What characteristics can be determined from this equation?
2. Determine the velocities and accelerations of the motions described by the following equations: $X(t) = 3 + 5t$, $X(t) = 4 - 2t$, $x(t) = 3 + 4t^2$, $x(t) = 6 - 2t + 3t^3$.
3. What is the difference between displacement and distance?
4. What is the difference between speed and velocity?
5. What is a material point? Provide examples when a human being can be considered as a material point.
6. What is the principle of relativity?
7. A boat moves in a river from point A to point B with a velocity $V = 5$ km/h relative to the water. The width of the river is $L = 4$ km, and the velocity of the water flow is $U = 3$ km/h along the shore. Using the law of addition of velocities and the principle of independence of motion along the x and y axes, determine the velocity of the boat relative to the shore when reaching point B, the angle at which the boat should move to reach point B and the time required to cross the river.



8. How does the normal acceleration change if the angular velocity decreases fourfold?

9. How does the gravitational interaction between two planets change if the distance between them is halved?

10. How will the acceleration of free fall change when ascending from sea level to a height $h=10$ km? Radius of the Earth $R_3=6370$ k? Find and compare the acceleration of gravity on Earth and Mars.

11. How will the weight of a body change if its mass has increased fivefold?

12. How will the weight of a body change if a mass $m=10$ kg moves upward with an acceleration $a=2$ m/s²?

13. How will the weight of a body change if a mass $m=20$ kg moves upward with an acceleration $a=2$ m/s²?

14. A projectile with a mass of 100 kg strikes a stationary platform with a mass of 10 tons at a speed of 50 m/s. Determine the speed of the platform after the collision.

15. Find the traction force of a 50 kW motor, if the speed of the car is 20 m/s.

16. A ball of mass m , suspended from a thread of length l , is deflected by 90° and released. Determine the maximum tension in the thread.

17. A body moves around the Earth. What speed must it have to remain in orbit without falling to the Earth? Identify the forces acting on the body, state Newton's law, and calculate the centripetal acceleration. The radius of the Earth is $R=6400$ km.

18. A body rolls down a chute. Determine the height from which the body must be released to complete a vertical loop of radius R without falling at the highest point A. Use the law of conservation of energy and Newton's law.

19. A point mass is attached to a wall by a spring and can move along the horizontal axis x . The mass experiences an elastic force proportional to the displacement X of the spring. The spring has an elastic constant k , and X_0 is the coordinate of the point mass when the spring is undeformed. Write Newton's law for the point mass, express its potential energy, and calculate it for $X=20$ cm, $X_0=10$ cm, and $k=0.2$ N/m.

20. A cylindrical body with a mass of 2 kg and a radius of 2 m rolls down an inclined plane at an angle of $\varphi=45^\circ$. Identify the forces acting on the body. Calculate the acceleration of the body as it rolls, given a coefficient of friction $k=0.02$. Take into account the rotation of the body.

21. A person with mass $m_1=60$ kg stands on the edge of a horizontal platform. The platform, a round homogeneous disk with mass $m_2=120$ kg, rotates around a vertical axis through its centre with a frequency $\nu=0.1$ Hz. Determine the platform's rotation frequency in revolutions per minute if the person moves from the edge to the centre. Write the law of conservation of angular momentum. Treat the person as a material point.

22. A stone is thrown vertically upward with an initial kinetic energy of 50 J. What is its potential energy at the highest point of its trajectory if air resistance is negligible?

23. An athlete rotates on a platform with an angular velocity of 4 rad/s. The athlete can be represented as a homogeneous cylinder with a radius $r=0.3$ m, height $h=1.7$ m and mass $m=70$ kg. After stretching out their arms, their moment of inertia changes and the rotation speed decreases to 2.8 rad/s. Determine the athlete's moment of inertia. State the law of conservation of angular momentum.

24. With what speed will a ball falling from a height of 5 m hit the ground?

25. What pressure does a 5 m high marble column exert on the ground?

26. An iron ruler 1 m long is cooled from 15°C to -35°C . Calculate the change in its length.

27. How will the volume of gasoline change if the temperature increases by 20°C ?

28. Find the force required to stretch a 2 m long steel wire by 1 mm if the cross-sectional area of the wire is 0.5 mm^2 .

29. How will the surface tension coefficient of a liquid change when the length of the vessel contour is halved; when the surface area is increased twofold?

30. How does the height of a liquid column in a capillary change if the liquid's density is halved; if the tube's radius is increased threefold?

31. For which type of meniscus is the Laplace pressure greater: convex or concave?

32. What determines the pressure inside a liquid?

33. At what depth does the water pressure in the sea equal 412 kPa?

34. Water is poured into an elbow of communicating vessels, and mercury into the other. What is the height of the mercury column if the height of the water column is 32 cm?

35. If the base area of a vessel is twice that of the other and they are filled with water to the same height, in which of them will the pressure at the bottom be greater?

36. Two vessels with the same base area — a cylindrical and a conical one — are filled with the same volume of water. In which vessel will the pressure at the bottom be greater?

37. What is the nature of atmospheric pressure, and how does it change with a variation in height?

38. How does the pressure on a scuba diver change with depth, assuming the liquid's density remains constant?

39. Will the response of a dynamometer change if a ball suspended from it is submerged in water? Is the buoyant force acting on the ball the same if its body is submerged into a different medium?

40. Will the buoyant force on a balloon change if the room air is heated by 10°C ?

41. Which is harder to hold in water — a brick or a piece of iron — if they have the same mass?

42. Two balls, one lead and one aluminium, are balanced on a lever in air. How does their balance change when they are submerged in water?

43. Two identical balls are balanced on a lever in air. What is the relationship between them if one of them is submerged in water and the other in kerosene?

44. Where and why is the draft of a ship greater — in a river or at sea?

45. How to determine the mass of a body in a liquid?

46. Why is the buoyant force negligible when an object is exposed to air?

47. What criteria are used to determine types?

48. How will the volume of a flowing liquid change if the volume of a pipe is doubled?

49. How does the speed of a liquid flow change if its viscosity is doubled?

50. How does the hydraulic resistance change if the liquid's viscosity is halved?

51. What parameters determine viscosity?

52. Why is the force of internal friction transmitted by coolants?

53. Explain the dependence of viscosity and mean free path on temperature?

Topic 2. Mechanical vibrations and waves. Acoustics

1. What is the difference between free vibrations and forced vibrations?

2. How can damped vibrations be made undamped?

3. How will the period of a pendulum change if the rope length is doubled and a 2 kg load is added to the board?

4. How will the period of a pendulum change if its frequency is reduced by two-thirds?

5. For which ball suspended on a spring — aluminium or iron — will the period of vibration be greater if their diameters are the same?

6. How can the angular frequency and period be determined if the linear frequency is known?

7. Determine the amplitude of vibrations, frequency and phase if the equation of vibrations is $x = 8 \sin(5t + 0.3)$.

8. Write the equation of harmonic vibration if the amplitude is 10 cm, the period is 10 s, and the initial phase is zero.

9. What is the difference between transverse and longitudinal waves?

10. What type of waves are the waves emanating from the sound of a bell and the waves on the sea surface?

11. What travels faster in water — sound or smell?

12. How is determined the speed of wave propagation determined in a medium?

13. What parameters of a medium determine the speed of wave propagation? Which characteristics of a wave change when it passes from one medium to another?

14. In what medium does sound not propagate and why?

15. Compare the period and frequency of a ball on a string that makes 60 oscillations in 2 minutes with a spring pendulum that completes 16 oscillations in 4 s.

16. How many full oscillations will a pendulum make in 10 s if the oscillation frequency is 220 Hz?

17. A ball on a string moves from its leftmost position to its rightmost position in 0.1 s. Determine the period of its oscillations.

18. How will the length of a sound wave change when the frequency of its source oscillations is halved?

19. When measuring a person's pulse, 150 beats were recorded in 2 minutes. Determine the frequency of the heart's contractions.

20. The upper limit of the oscillation frequency perceived by the human ear is 22 kHz for children and 10 kHz for elderly people. If the speed of sound is 340 m/s, will a sound with a wavelength of 20 mm be heard only by a child, or by an elderly person as well?

21. A person perceives frequencies from 20 Hz to 20 kHz. What range of wavelengths corresponds to the audibility interval if the speed of sound is 340 m/s?

22. What is the difference between travelling and standing waves?

23. What are harmonic, mode and overtone?

24. What is the acoustic spectrum of a complex tone?

25. What characteristic of vibrations determines the volume, timbre and pitch of a sound?

26. What changes are perceived by a person when the amplitude and frequency of sound vibrations increase?

27. How are the volume and intensity of a sound wave related?

28. What is the source of sound, ultrasound and infrasound waves?

29. What effect do ultrasound and infrasound have on a biological organism?

30. Name the applications of ultrasound.

Topic 3. Molecular physics. Thermodynamics

1. What is the physical significance of Avogadro's number?

2. How can the mass of a single molecule be determined?

3. How can the density of a substance be determined?

4. What are normal conditions?

5. Explain Avogadro's law.

6. What are thermodynamic macroparameters?

7. What is the difference between open, closed and isolated thermodynamic systems?

8. What is the physical meaning of the Boltzmann constant, and how does it relate to the gas constant?

9. Describe the relationship between the speed of movement, kinetic energy of molecules and temperature?

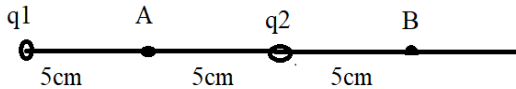
10. By what factor will the kinetic energy of particles change if the body's temperature is doubled?
11. What is absolute zero, and what is its physical significance?
12. What law determines the relationship of thermodynamic macroparameters?
13. What does the Clapeyron formula determine?
14. What are isoprocesses and gas laws?
15. Do the gas laws apply if the gas is in an open vessel?
16. What factors determine gas pressure?
17. What is partial pressure?
18. What does Dalton's law describe?
19. What equation defines the behaviour of a real gas?
20. If the air in a room is heated by 150°C , how will its volume and mass change?
21. How will the pressure in a closed vessel change if the volume has doubled and the process is isothermal?
22. How will the volume of gas in a closed vessel change if the temperature decreases threefold, assuming the process is isothermal?
23. How does the concentration of molecules change with a change in the volume of steam?
24. How does steam pressure depend on temperature?
25. How does the boiling point depend on pressure, and what factors determine the boiling time?
26. At what temperature can a gas no longer be converted into a liquid?
27. What happens when a gas is cooled below the dew point?
28. How are relative humidity and absolute humidity determined?
29. What is the mechanism of melting and crystallisation in substances?
30. What are the components of the internal energy of a body?
31. Will the internal energy change if the mass of a body (gas) increases, if its volume increases, if its temperature increases, or if its kinetic energy increases?
32. What are the ways to change a body's internal energy?

33. What is the difference between the types of heat transfer?
34. What is the physical meaning of heat transfer, and by what physical quantity is it determined?
35. How does energy conversion occur during heat exchange? What illustrates the fulfilment of the heat balance equation?
36. What is the difference between heat capacity and specific heat capacity?
37. How is the work of a gas determined? In which case is it equal to zero?
38. How to write the first law of thermodynamics if no work is done or if no heat exchange has occurred?
39. Why do the formulas for phase transformations not include temperature change?
40. Under what conditions can phase transition processes occur?
41. What is the mechanism of evaporation and condensation, and on what factors does the evaporation rate depend?
42. What are dynamic equilibrium and saturated steam?
43. What is the difference between crystalline and amorphous bodies?
44. What media are considered isotropic?
45. What is polymorphism?
46. How will the internal energy of a gas change during an isothermal process if the pressure has increased threefold?
47. What work will a gas perform during an isochoric process if its temperature has increased fourfold?
48. What amount of heat is required to raise the temperature of ice from -15°C to 30°C ?
49. Find the amount of heat required to melt a piece of ice at -10°C and then evaporate the resulting water.
50. How to find the amount of heat released during the combustion of 100 g of gasoline?

Topic 4. Electrodynamics. Magnetism

1. What is the Coulomb force, and what factors determine it?
2. What is an elementary charge? Provide examples of elementary particles.

3. How can a body be charged positively or negatively?
4. Explain the law of conservation of charge.
5. What is the nature of an electrostatic field? Give formulas for the force and energy characteristics of an electric field.
6. What are lines of tension and their properties? Draw the lines of force for a point charge and a dipole.
7. What is the principle of superposition?
8. Explain Coulomb's law. Calculate the electric field strength at points A and B, which are located at a distance from two charges, $q_1 = -5\text{C}$ and $q_2 = 4\text{C}$, lying on a straight line as shown in the figure. Indicate the directions of the electric fields at points A and B produced by charges q_1 and q_2 .



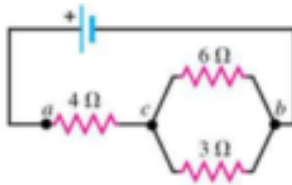
9. What is Gauss's theorem? Using Gauss's theorem, find the electric field strength at a distance of 30 cm outside and 10 cm inside a charged sphere of radius $R = 20\text{ cm}$, if the centre of the sphere contains a charge $q_1 = 2\text{ }\mu\text{C}$, and the charge is uniformly distributed on the surface with a density of $-5\text{ }\mu\text{C}/\text{m}^2$.
10. What is the difference in the characteristics of the field inside and outside a charged sphere?
11. How can electrostatic protection be provided for a device?
12. What are potential fields, and how does the work of a field depend on the shape of the trajectory?
13. What is the property of equipotential surfaces?
14. How will the electric field strength and potential change if the distance from the charge is doubled?
15. What is the physical significance of permittivity?
16. What characterises electrical capacitance; with changes in what parameters does it change?

17. Into which groups can substances be classified based on their electrical conductivity? What distinguishes the electrical conductivity of metals, electrolytes and gases?

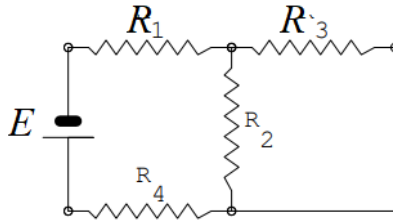
18. What is electric current? What is the difference between direct and alternating current?

19. Describe the effects of electric current.

20. Write Ohm's law for a complete circuit if the EMF is 20 V and the internal resistance $r_{EMF}=2\ \Omega$. Calculate the current in the circuit.



21. Explain Ohm's law. Apply Ohm's law to a circuit. Calculate the current I in a circuit where $R_1=3\ \Omega$, $R_2=4\ \Omega$, $R_3=5\ \Omega$, $R_4=4\ \Omega$, and $E=6\ \text{V}$.



22. Explain Ampère's law. Why, according to Ampère's law, do two parallel conductors with currents in the same direction attract each other, while two parallel conductors with currents in opposite directions repel each other? Indicate the direction of the forces acting on the current-carrying conductors.

23. Provide the formulas for electric current and current density, including the expressions specific to metals.

24. What are external forces and EMF? What characterises voltage? Under what conditions does electric current occur, and why is it absent in a conductor when no electric field is applied?

25. What is the law of electromagnetic induction, and what is Lenz's law? Write the formula that expresses the law of electromagnetic induction.

26. What limits the speed of electrons in metals?

27. What determines the value of resistance?

28. How does the electrical resistivity of metals change with temperature? What is superconductivity?

29. What is the temperature coefficient of resistance? Why does electric current decrease when an electrolyte is cooled?

30. Which law determines the relationship between current characteristics, and why were two forms of the law introduced?

31. Under what conditions can a liquid become a conductor?

32. Write Ohm's law for a circuit containing resistance, capacitance and inductance. What is the total impedance? Under what conditions does resonance occur, and what is the resonant frequency?

33. What is the combined law of electrolysis?

34. What determines the electrochemical equivalent?

35. Under what conditions is electrical conductivity possible in semiconductors?

36. What is the difference between p-type and n-type semiconductors?

37. What is the difference between donor and acceptor impurities?

38. Why do dielectrics not conduct electric current?

39. How does the force of interaction between charges change when immersed in a dielectric?

40. What is the difference between polar and non-polar dielectrics?

41. Describe the nature of magnetic fields and the methods for detecting them.

42. How can the magnitude and direction of a magnetic force be determined?

43. How is induction determined for different conductor configurations?
44. What is the distinctive feature of a vortex field?
45. What is the Ampère force, and when does it reach its maximum value?
46. What explains the interaction of two parallel conductors with direct current?
47. What is the Lorentz force, and how can its direction be determined when it is zero?
48. How to write the equation of motion of a charged particle in a magnetic field?
49. What characterises the magnetic permeability of a substance?
50. Explain the motion of a charged particle with velocity v in a magnetic field B perpendicular to its velocity. What determines the radius of its trajectory?
51. A current-carrying frame is placed in a uniform, constant magnetic field. How will the frame behave in the field? Indicate the forces acting on the sides of the frame. Calculate the torque if the magnetic field is 2 mT, the side of the frame is 5 cm, and the current is 2 mA.
52. What types of magnets exist? Describe the behaviour of magnetic materials in a magnetic field.
53. What is the difference between dia-, para- and ferromagnets? What happens when a magnetic material is heated above the Curie temperature?
54. What is electromagnetic induction?
55. Under what conditions does an induced current arise?
56. Under what conditions does the rate of change of magnetic flux increase?
57. What is self-induction?
58. How can the energy of a magnetic field be determined? Which mechanical quantity is analogous to inductance?

Topic 5. Optics

1. What is the duality of the nature of light?
2. What is an electromagnetic wave, and what characteristics define it?
3. What are the main properties of electromagnetic waves?
4. Name the ranges of the EMW scale.
5. What is light, what is the range of wavelengths of the light range on the electromagnetic wave scale?
6. What lengths of monochromatic waves are included in the spectrum of white light?
7. What is the speed of electromagnetic waves in a vacuum and matter?
8. How will the wave's speed and frequency change when it enters a medium with a refractive index twice as large?
9. What photometric quantities characterise the luminous flux?
10. What is the difference in how the eye perceives monochromatic waves?
11. What phenomenon is called interference, and under what conditions does it occur?
12. On what factors does the width of an interference fringe depend? Explain why a rainbow interference pattern appears on the screen when white light is used in Young's experiment, whereas alternating red and black fringes appear when monochromatic red light is used.
13. Under what conditions does the maximum and minimum intensity of light occur during interference?
14. Describe the methods for obtaining coherent sources used to observe light interference.
15. Under what conditions does the phenomenon of diffraction occur?
16. What is the difference between Fresnel and Fraunhofer diffraction?
17. Under what conditions are diffraction intensity maxima observed when using a diffraction grating?

18. What determines the Rayleigh criterion?
19. What is dispersion?
20. Which media are anisotropic?
21. What is polarisation, and what are the different types of polarisation?
22. What is Malus's law?
23. What is Fermat's principle? Show how the law of refraction of light can be derived from this principle.
24. Formulate the laws of geometric optics.
25. Explain the causes of mirages observed in deserts.
26. Under what conditions can a clear image be obtained in lenses?
27. What is the relationship between optical power and focal length?
28. What quantities are related by the thin lens formula?
29. How can the magnification factor of a lens be calculated?
30. What are visual aberrations?
31. What is a diffraction grating? What determines the resolution of a diffraction grating? What is the minimum number of slits required for a diffraction grating to resolve two violet lines in the hydrogen spectrum, if the wavelengths are $\lambda_{\text{violet}1} = 434.0 \text{ nm}$ and $\lambda_{\text{violet}2} = 410.1 \text{ nm}$?

Topic 6. Quantum physics

1. How are the mass, momentum, wavelength and energy of a photon determined?
2. What is the Rutherford model of the atom? Why is Rutherford's model of the atom not valid? Can any atomic model explain the spectra of atoms?
3. When does an atom become ionised?
4. What are the characteristics of the neon isotopes with mass numbers 20, 21 and 22?
5. What is the difference between emission and absorption spectra of light?

6. Which substances produce continuous, line and band spectra?
7. Formulate Bohr's postulates.
8. Describe the spectral series of a hydrogen atom.
9. Describe the properties of the particles that constitute an atom and its nucleus. Do atomic nuclei exist with a charge smaller than that of a proton?
10. How can the number of neutrons and protons in a nucleus be determined?
11. What are the characteristics of an atomic nucleus? What determines the stability of atomic nuclei?
12. How is binding energy determined?
13. How is the mass defect calculated?
14. What factors contribute to the natural radioactive background of the Earth?
15. By what indicators is radioactive radiation detected?
16. What is the difference between alpha, beta and gamma radiation, and how can protection against them be ensured?
17. How does radioactive radiation affect living organisms?
18. Formulate the law of radioactive decay.
19. How can the number of decayed nuclei be determined?
20. How can the half-life of a substance be determined?
21. How is the rule for the change in nuclei during radioactive decay expressed?
22. What is the difference between exothermic and endothermic reactions?
23. What is the photoelectric effect?
24. Does the speed of electrons depend on the wavelength of incident light during the photoelectric effect?
25. Under what condition is the red border of the photoelectric effect observed?
26. What physical quantities determine light pressure?
27. What is thermal radiation? Explain the Stefan — Boltzmann law and Wien's law?
28. What are absolute black bodies? What model do physicists use to represent a black body?

29. Explain Max Planck's fundamental model used to describe the laws of thermal radiation. Write the Planck function and sketch the corresponding graph.

30. What factors determine the reflection coefficient? Why do dark objects heat up more in sunlight?

31. The temperature of the first body is lower than that of the second. For which body does the maximum radiation correspond to a shorter wavelength?

32. The maximum radiation is at 450 nm for one body, and at 550 nm for the other. Which of the bodies has a higher temperature?

VII. ASSIGNMENTS FOR INDEPENDENT STUDY

Topic 1. Mechanics

1. The speed of a car increases from 18 km/h to 61.2 km/h in 2 minutes. What is the car's acceleration?

2 Find the force acting on a body with a mass of 300 g that travels 25 m in 5 s.

3. A tourist first walked 4.5 km north, then 6 km south. Determine the magnitude of the tourist's displacement.

4. A body moves with a constant acceleration of 5 m/s^2 . Determine the distance it travels during one second of motion if its initial velocity is zero.

5. A rocket with a mass of 20 000 kg takes off with an acceleration of 25 m/s^2 . Determine the rocket's thrust.

6. A car with a mass of 250 kg travels 200 m along a horizontal road. Determine the work done by gravity.

7. A spring gun is fired vertically downwards at a target located 2 m away. The bullet transfers 0.12 J of energy to the target as it comes to a stop. What is the mass of the bullet if the spring was compressed by 2 cm before the shot and has a stiffness of 100 N/m?

8. What work must be done to lift a body weighing 75 kg to a height of 2 m?

9. What work must be done to stretch a spring with a stiffness of 40 kN/m by 5 cm?

10. A boy weighing 50 kg, running at 4 m/s, jumps onto a cart weighing 100 kg moving towards him at 3 m/s. Determine the speed of the cart with the boy on it.

11. What work must be done to dig a hole in the ground with an area of 4 m^2 and a depth of 1 m, if the soil has a density of $5 \times 10^3 \text{ kg/m}^3$?

12. A body falls freely from a height of 40 m. Determine the speed of the body as it reaches the ground.

13. A person weighing 50 kg sits at the stern of a 3 m-long boat weighing 100 kg. How far will the boat move if the person walks from the stern to the bow, assuming negligible friction with the water?

14. A cannonball explodes into three equal fragments, all moving in a horizontal plane. The first fragment flies north at 2 m/s, and the second flies west at the same speed. Determine the speed of the third fragment.

15. A body is thrown vertically upward at a speed of 20 m/s. At what height does the kinetic energy of the body equal its potential energy?

16. A car moving at a speed of 80 km/h suddenly brakes. Determine the length of the braking distance if the coefficient of friction between the wheels and the road is 0.6.

17. A boy weighing 50 kg floats on an ice floe with an area of 4 m². What is the minimum thickness of the ice floe required to support him?

18. At what height above the Earth's surface will a 9 kg body have a weight of 10 N?

19. At what depth in fresh water is the pressure three times the atmospheric pressure (1×10^5 Pa)?

20. Equal masses of mercury and water are poured into a cylindrical vessel. The total height is 30 cm. Find the pressure at the bottom of the vessel.

21. Water is poured into two identical tanks, each 1 m high. In one tank, the tap is 15 cm from the bottom; in the other, it is 0.25 m from the bottom. In which tank will the water level drop to 50 cm faster?

22. Determine the excess pressure in an air bubble with a diameter of 20 μm in water ($\sigma = 75$ mN/m).

23. A person holding a barometer slowly rises from the Earth's surface. At what height (in metres) will the barometer reading change by 2 Pa?

24. At what height will the atmospheric pressure decrease by a factor of ten?

25. Determine the height of water rise in a capillary with a diameter of $5\ \mu\text{m}$, assuming ideal wetting. The surface tension coefficient is $75\ \text{mN/m}$.

Topic 2. Oscillations and waves

1. Find the length of a sound wave if its speed in the air is $340\ \text{m/s}$, and the frequency of the source is $200\ \text{Hz}$.

2. The frequency of a crow's wing oscillations is $3\ \text{Hz}$. How many wingbeats will the crow make while flying $650\ \text{m}$ at a speed of $13\ \text{m/s}$?

3. The distance between the crests of the waves in the sea is $6\ \text{m}$. Find the period of the waves hitting the hull of a boat if their speed is $3\ \text{m/s}$.

4. What is the length of a wave propagating along a cord at a speed of $6\ \text{m/s}$ if its frequency is $4\ \text{Hz}$?

5. A tuning fork produces a sound wave $0.5\ \text{m}$ long. What is the frequency of the oscillations if the speed of sound is $340\ \text{m/s}$?

6. A man hears thunder $10\ \text{s}$ after seeing a lightning flash that struck $3.3\ \text{km}$ away. Determine the speed of sound.

7. The echo of a gunshot reaches the shooter after $2\ \text{s}$. Determine the distance to the obstacle that reflected the sound, given that the speed of sound is $340\ \text{m/s}$.

8. A sonar signal was reflected from a target located $3\ \text{km}$ away and was recorded $4\ \text{s}$ after its return at a frequency of $10\ \text{kHz}$. Find the length of the sound wave in water.

9. What is the distance between a ship and an iceberg if an ultrasonic sonar signal sent from the ship is received back after $4\ \text{s}$, given that the speed of sound in water is $1500\ \text{m/s}$?

10. The amplitude of oscillations of a point on a string is $2\ \text{mm}$, the frequency is $1\ \text{kHz}$. What distance will the point travel during the oscillation period and in $0.4\ \text{s}$?

11. Write the equation of harmonic oscillation if the total energy is 0.3 kJ, the maximum force is 1.5 N, the oscillation period is 2 s, and the initial phase is 600° .

12. Write the equation of harmonic oscillation with an amplitude of 5 cm, a period of 4 s, and an initial phase of 450. Find the speed and acceleration.

13. A material point oscillates with a period of 2.4 s, an amplitude of 5 cm, and an initial phase of 0. Determine its displacement, velocity and acceleration.

14. A body with a mass of 2 kg oscillates according to the law $x=50 \sin(\pi/3 t)$. Determine the amplitude and total energy of the oscillations.

15. Find the period of oscillation of a spring pendulum with a mass of 200 g, given that the elasticity coefficient is 200 N / m.

16. Oscillations propagate according to the law $x=0.05 \sin 500 \pi t$. Find the displacement of a point at a distance of 60 cm from the source 0.2 s after the start of the oscillations, given that the propagation speed is 300 m/s.

17. A material point performs harmonic oscillations with a period of 2 s, an amplitude of 50 mm and an initial phase=0. Find the velocity of the point at the moment when the displacement reaches 25 mm.

18. The oscillatory circuit consists of a coil with an inductance of 0.003 H and a capacitor with a capacity of 13.4×10^{-9} F. Determine the period of free oscillations in the circuit.

19. A coil with an inductance of 0.08 H is connected to a source of alternating voltage with a frequency of 1000 Hz. Find the capacitance of the capacitor in the circuit.

20. A voltage applied to a coil changes over time according to the law $U=30 \cos (10 \pi t)$. Determine the inductance of the coil if the effective value of the current flowing through it is 5 A.

21. A coil with an inductance of 0.001 H and a capacitor with a capacitance of 5 μ F are connected to an AC circuit. At what frequency does the current oscillate?

22. How must the inductance of a coil in an oscillatory circuit change if the oscillation frequency increases from 400 Hz to 500 Hz, given that the capacitor has a capacitance of 10×10^{-6} F?

23. In an ideal oscillatory circuit, the amplitude of the voltage across the capacitor is 8 mV, and the amplitude of the current in the coil is 2.0 mA. At a certain time t , the current in the coil is 1.2 mA. Determine the voltage across the capacitor at time t .

24. When a coil is connected to a DC circuit with a voltage of 12 V, the ammeter shows a current of 4 A. When the same coil is connected to an AC circuit with a frequency of 50 Hz and a voltage of 12 V, the ammeter shows 2.4 A. Determine the inductance of the coil. What will be the active power of the current in the circuit if a capacitor with a capacitance of 394 μ F is connected in series with the coil?

25. Resonance in an oscillatory circuit with a capacitor of $C_1 = 1 \mu$ F occurs at a frequency of $\nu_1 = 400$ Hz. When another capacitor C_2 is connected in parallel to C_1 , the resonant frequency becomes $\nu_2 = 100$ Hz. Determine the capacitance of C_2 .

Topic 3. Molecular kinetic theory and thermodynamics

1. How many molecules are contained in 1 kg of hydrogen?
2. Find the mass of a molecule of ozone, hydrogen, carbon dioxide and methane.
3. Determine the number of air molecules in 1 cm³ of a vessel at 10 °C and a pressure of 1.5 μ Pa.
4. How many water molecules are in a drop weighing 0.2 g?
5. Determine the density of hydrogen at 150 °C and a pressure of 90 kPa.
6. The mass of alcohol at 0 °C in a volume of 500 cm³ is 400 g. Determine the density of alcohol at 15 °C.
7. The average kinetic energy of gas molecules at 1000 °C is 1.6×10^{-23} J. Determine the energy at 500 °C.
8. Gas flows out of a 10 L cylinder. The pressure gauge reads 5 Pa at 7 °C and 17 °C. Determine how much gas has leaked.

9. Determine the concentration of oxygen molecules if the pressure is 0.2 MPa and the root-mean-square speed of the molecules is 700 m/s.

10. A vessel with a capacity of $3 \times 10^{-3} \text{ m}^3$ is filled with nitrogen under a pressure of $2 \times 10^5 \text{ Pa}$ at a temperature of 27°C . Determine the mass of nitrogen in the vessel if its molar mass is 0.028 kg/mol.

11. How will the volume and mass of air in a room change when it is heated by 10°C ?

12. How will the kinetic energy of particles change when the temperature of a body is doubled?

13. At what temperature does the root-mean-square velocity of oxygen molecules equal 500 m/s?

14. Find the pressure of 1 mole of gas molecules occupying a volume of 1 litre at a temperature of 300 K.

15. Carbon dioxide is heated from 20°C to 30°C at constant pressure. Determine the work done by the gas and the change in its internal energy.

16. A 1 L vessel contains 12 g of oxygen. Determine the pressure at 15°C .

17. Fifty grams of ice at -15°C are added to a vessel containing 100 g of water at room temperature. Determine the final equilibrium temperature.

18. Ice weighing 0.5 kg at -10°C is placed into 2 litres of water at 70°C . Determine the final equilibrium temperature.

19. How much ice will melt if 5 kg of ice at 0°C is placed into 10 kg of water at 0°C ? Provide the answer in kg.

20. What amount of heat is released during the combustion of 10 kg of kerosene, if the specific heat of combustion of kerosene is 46 MJ/kg? Provide the answer in MJ.

21. What amount of heat is required to melt a piece of ice taken at -10°C and to evaporate the resulting water?

22. Find the amount of heat required to melt a piece of aluminium wire 20 cm long and 1.5 mm thick, taken at room temperature.

23. Find the amount of heat required to evaporate 2 litres of water at room temperature.

24. Nitrogen expands isochorically, changing pressure from 202 kPa to 101 kPa. Determine the work done by the gas and the change in internal energy if the initial temperature is 15 °C.

25. Nitrogen expands isothermally at 200 °C, changing pressure from 202 kPa to 101 kPa. Calculate the change in internal energy.

26. When a gas is compressed, its volume decreases from 8 L to 5 L, and the pressure increases by 60 kPa. Find the initial pressure.

27. A vessel contains 1 kg of ice at -20°C , 0.5 L of water at 0°C , and boiling water. Write the heat balance equation.

28. A 0.5 kg block of ice at -5°C is placed in a 200 g calorimeter containing 0.5 litres of water at 50°C . How can the specific heat capacity of the calorimeter be determined if the final equilibrium temperature is 2°C ?

29. A lead bullet flies at a speed of 200 m/s. How will its temperature change if all its kinetic energy is converted into heat?

30. A bullet fired vertically upward reaches a height of 1200 m. How much does it heat up upon hitting the ground if half of its energy is converted into heat?

Topic 4. Electrodynamics

1. What is the force acting at a distance of 5 cm between charges of 6 and 12 nC?

2. Find the potential of a sphere with a radius of 3 cm if the charge of the sphere is 1.5 nC.

3. Determine the capacitance of series-connected capacitors of 2, 4 and 6 μF .

4. The distance between two stationary point charges is increased by a factor of 3. How does the force of interaction between the charges change?

5. How will the force of interaction between two identical point charges change if half the charge of one body is transferred to the second body?

6. The electric field strength of a dipole is 20 V/m at a distance of 2 m. Determine the field strength at a distance of 4 m.

7. Two point charges $q_1 = -2 \text{ nC}$ and $q_2 = 18 \text{ nC}$ are separated by 40 cm. At what distance from the first charge is the electric field zero?

8. Two identical metal balls have charges $q_1 = 2 \text{ nC}$ and $q_2 = -6 \text{ nC}$. The balls are briefly brought into contact and then separated to the same distance. How many electrons are transferred to the first ball during contact?

9. Two balls with equal charges of 6 nC are initially far apart; the diameter of the first is twice smaller than that of the second. The balls are briefly brought into contact and then returned to their original positions. What is the charge of the first ball?

10. How will the resistance change if the cross-sectional area is reduced by a factor of 3?

11. What charge is required for a current of 0.1 A to pass through a conductor in 10 s?

12. Calculate the resistance of a section of a circuit with a current of 12 A and a voltage of 120 V.

13. The resistance of a resistor is doubled, and the applied voltage is halved. How does the current across the resistor change?

14. Determine the current in a circuit consisting of a source with EMF 12 V and internal resistance 3Ω connected to a resistor of 3Ω .

15. Determine the resistance of a resistor if the amplitude value of the current flowing through it is 3.1 A, and the effective value of the voltage is 220 V.

16. Two conductors made of the same material, equal in length, but with different cross-sections ($S_1 > S_2$) are connected in series in a circuit. Compare the electric field strengths in the first and second conductors (E_2/E_1).

17. When the battery terminals are short-circuited, the current in the circuit is 20 A. When an electric lamp with a filament resistance of 5.4Ω is connected to the battery terminals, the current in the circuit is 2 A. Determine the EMF and internal resistance of the battery.

18. Determine the heat released in a conductor with a current of 2 A and resistance of 0.5Ω over 10 s.

19. A direct current flows through a homogeneous copper cylindrical conductor 40 m long. Determine the potential difference if the conductor heats by 16 K in 15 s. Neglect changes in resistance and heat losses during heating. Copper density is 8890 kg/m^3 , resistivity is $1.724 \times 10^{-8} \Omega \cdot \text{m}$, and specific heat capacity is $0.375 \text{ J/(g} \cdot \text{K)}$.

20. A conductor carries a current of 0.12 A under a voltage of 12 V. How will the current change if the voltage is doubled?

21. A copper wire of length L and diameter d has voltage U applied across it. If the length of the conductor is doubled without changing the voltage, how will the average speed of ordered electron motion along the conductor change?

22. A resistor of 4Ω is connected to a source with internal resistance 2Ω , and 96 W of power is delivered to the load. Determine the maximum power that can be obtained from the source.

23. The electrochemical equivalent of copper is 0.33 mg/C . What amount of copper will be released in 1 hour at a current of 40 A?

24. Determine the magnitude of the Lorentz force on a charge of $2 \mu\text{C}$ moving at 500 km/s perpendicular to a magnetic field of 20 mT.

25. Determine the magnitude of the magnetic induction vector if a force of 10 N acts on a charge of $5 \mu\text{C}$ moving at 500 km/s perpendicular to the vector.

26. In a uniform magnetic field with an induction of 60 T, a proton moves perpendicular to the induction vector with a speed of 30 m/s. Determine the radius of the proton's trajectory.

27. An electron flies into a uniform magnetic field with an induction of 40 T perpendicular to the induction lines of this field and moves in a circle with a radius of 4 m. Calculate the speed of the electron.

28. A rectilinear conductor 10 cm long is in a uniform magnetic field with an induction of 4 T and is located at an angle of 30° to the magnetic induction vector. Determine the force acting on the conductor from the magnetic field if the current in the conductor is 3 A.

29. A rectangular wire frame made from 8 cm of wire carries a current of $I=20$ mA and is placed in a uniform magnetic field with an induction of $B=0.2$ T. Determine the maximum Ampère force acting on the frame.

30. An electron flies into the space between two oppositely charged plates of a flat capacitor with a velocity V parallel to the plates. The distance between the plates is d , the length of the plates is l , and the potential difference between the plates is $\Delta\varphi$. Determine the angle by which the electron will deviate after leaving the capacitor.

Topic 5. Optics and quantum physics

1. Determine the ratio of the thicknesses of diamond and glass plates if the time of light propagation through them is the same.

2. What is the magnification of a magnifying glass with an optical power of 16 dioptres?

3. What is the optical power of a system of lenses if one has an optical power of 16 dioptres and the other has a focal length of 9 cm?

4. At what distance from a converging lens will an image magnified three times be located if the object is 20 cm away?

5. A converging lens gives a real image of the object, magnified by two times, the distance from the object to its image is 18 cm. Determine the focal length of the lens.

6. The angle of incidence of a ray from air onto a flat surface of a liquid is 54° , and the angle of refraction is 30° . Determine the relative refractive index of the liquid.

7. At what distance from a converging lens with focal length F should an object be placed so that the distance between the object and its image is minimal?

8. Determine the focus of a lens that magnifies an object two times if the distance between the lens and the image is 24 cm.

9. Determine the magnification of a microscope if the focal lengths of the objective and eyepiece are 3 mm and 50 mm, respectively, and the distance between them is 135 mm.

10. How does the focus of a lens change if it is placed in a medium with a refractive index twice as high?

11. Determine the angle of refraction if a beam of light falls from glass into water at an angle of 45° .

12. An object is located at a distance of 60 cm from a thin converging lens with a focal length of 20 cm. Determine the distance from the lens to the image.

13. A plano-convex lens with a focal length of 10 cm is to be made from glass with a refractive index of 1.5. Determine the radius of curvature of its spherical surface.

14. An object is placed 100 cm from a screen. Using a converging lens, two images of the object are obtained on the screen at two different lens positions, 20 cm apart. Determine the focal length of the lens.

15. A 4 m-high pile is driven into the bottom of a lake, with 1 m protruding above the water. Determine the length of the pile's shadow on the lake bottom if sunlight strikes the water surface at an angle of 45° . Take the refractive index of water as 1.4.

16. A photon with a wavelength of 50 nm knocks electrons with an energy of 7.0 eV out of a metal. Determine the work function of electrons for this metal.

17. Determine the maximum speed of an electron emitted from cesium (work function $A_{\text{out}} = 2$ eV) when illuminated with light of wavelength 400 nm. Take the electron mass $m_e = 9.1 \times 10^{-31}$ kg.

18. What are common characteristics of isotopes of a single element, such as hydrogen (mass numbers 1, 2, 3) and neon (mass numbers 20, 21, 22)?

19. What is the composition of the nucleus of: silver Ag^{107}_{47} , radium Ra^{226}_{88} , germanium Ge^{73}_{32} , niobium Nb^{93}_{41} , fluorine F^{10}_9 , uranium U^{238}_{92} , sodium Na^{23}_{11} , curium Cm^{247}_{96} and lead Pb^{207}_{82} ?

20. How many β -decays occur during a transformation into $^{214}_{83}\text{Bi}$ and $^{206}_{82}\text{Pb}$?

21. What fraction of radioactive radium atoms decay in a time interval equal to one-third of its half-life, and in a time interval equal to 0.4 of its half-life?

22. The half-life of polonium is 138 days. After what period of time will 1 g remain from 1 kg of polonium?

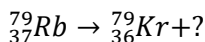
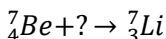
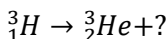
23. What fraction of the nuclei of a sodium isotope decays in 1 minute if its half-life is 14.8 hours?

24. Write the nuclear reaction that occurs when sodium ${}_{11}^{24}\text{Na}$ is bombarded with neutrons and a proton is emitted from the resulting nucleus.

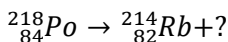
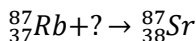
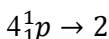
25. Write the nuclear reaction that occurs when aluminium is bombarded with alpha particles, producing a new nucleus and a neutron.

26. An unknown radioactive element decays according to the scheme $X = \text{Kr}_{36}^{91} + \text{Ba}_{56}^{142} + 2n$. Determine the number of neutrons in the unknown element.

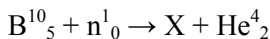
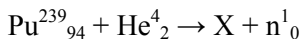
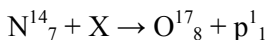
27. Determine which elementary particle is produced in the given nuclear reaction.



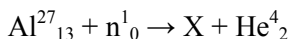
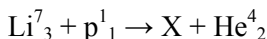
28. Determine which elementary particle is produced in the following nuclear reaction.



29. Complete the reactions:



30. Complete the reactions:



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Kupriyanova Galina Sergeevna
Molostova Svetlana Valerievna
Nestereva Zhanna Yurievna

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